Features of electron-spin-resonance excitation in impure asymmetric two-dimensional structures

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The band spin-orbit coupling, which makes it possible for the orbital motion of electrons to affect the spin dynamics, is known to be present in crystals with destroyed mirror symmetry. A framework for analyzing effects of the spin-orbit coupling on spin-dependent electron kinetics is suggested and applied to the electron-spin resonance on an electron gas in an impure asymmetric two-dimensional semiconductor structure. The general case of excitation of the resonance by both the electric and magnetic components of the microwave electromagnetic field is considered and the frequency-dependent tensors of the electric conductivity and spin susceptibility as well as the spin-current-correlation tensors, which additionally characterize the response of a broken-mirror-symmetry conducting media to an external electromagnetic field, are calculated. It is shown that the electric component of the resonant microwave field can excite the resonance more effectively than the magnetic component in spite of a small value of the spin-orbit coupling. The formalism presented allows one to consider the case when an external static magnetic field is arbitrary inclined with respect to the plane of the structure and the cyclotron frequency ω_c corresponding to the perpendicular component of the field can take any value less that the Fermi energy ϵ_F. It is found that the cyclotron motion not only modifies the spin relaxation time but also has an effect on the spin precession giving rise to a shift of the Larmour frequency. The shift can be positive or negative depending on the sign of g factor of current carriers relative to the sign of their charge. It is shown that due to the cyclotron motion the spin resonance can also take place in the particular case of zero g factor. It is also found that because of the spin-velocity correlations the absorption of the *linear* polarized radiation can change its value at the magnetic field reversal.

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I. INTRODUCTION

Electron spin resonance (ESR) has long been used to determine g factors of molecules and solids providing information about chemical structures of molecules and electron band structures of solids. In addition, the longitudinal and transverse relaxation times, T_1 and T_2, which can be inferred from the form of the absorption line, allow one to investigate interactions responsible for spin-flip transitions. Recently, the ESR has been employed to study g factors, g factor anisotropies, and spin relaxation in different two-dimensional (2D) semiconductor structures.[1–7] An enhanced attention paid to 2D structures is bound up with the growing interest in spintronic devices and quantum information processing[8] because a good understanding of the spin dynamics and processes responsible for destruction of spin coherence is vital to those fields. The ESR could help to attain all these objectives: it provides possibilities to investigate fundamental aspects of the spin dynamics by studying the ESR absorption and simultaneously with the help of an ESR pulse one can generate the spin magnetization of a system and control its state and evolution.[9] This paper aimed at an investigation of features of the ESR in 2D semiconductor structures connected with destroyed mirror symmetry.

As is known, the carrier-effective-mass Hamiltonian in such structures contains a term that couples the carrier's momentum and spin

$$H_{so} = \alpha(\mathbf{p} \times \mathbf{c}) \cdot \boldsymbol{\sigma}, \tag{1}$$

where α is a constant characteristic of the material, $\mathbf{c}$ is one of two nonequivalent normals to the plane of a structure, $\boldsymbol{\sigma}$ and $\mathbf{p}$ are, respectively, the Pauli spin and momentum operators, and units in which $\hbar = c = k_B = 1$ are used. The term, Eq. (1), can be considered as a consequence of parity violation with respect to reflection in the plane of the 2D electron layer. The spin-orbit coupling of this form was first introduced phenomenologically in Ref. 10 and microscopically in Ref. 11 for bulk crystals of polar symmetry; $\mathbf{c}$ is then the direction of the polar axis. (For the sake of uniformity, we shall call the vector $\mathbf{c}$ the polar axis both in the cases of bulk polar crystals and asymmetric 2D structures.) Hamiltonian (1) should be present in semiconductor quantum wells with wurtzite structure. Later its presence in some A_3B_5 semiconductor heterostructures,[12,13] surface states,[14] and silicon quantum wells due to lacking mirror symmetry of the confining potential[1–3,5] was confirmed. We shall term Hamiltonian (1) the band spin-orbit coupling (BSOC) to distinguish from the impurity spin-orbit coupling induced by scattering on heavy impurities. Usually, in the absence of the spin-orbit interaction, the orbital and the spin degrees of freedom of an electron evolve independently of each other. Hence, in the presence of a static magnetic field $\mathbf{B}^{(0)}$, the ESR, i.e., transitions between two states of an electron differing only in their spin orientation, can be excited by the magnetic component of an applied microwave field if a frequency of the field matches the electron Zeeman splitting $\omega_s = g\mu_B B^{(0)}$. Because of the BSOC, the spin degrees of freedom of an electron in a crystal are not independent on the orbital motion. Due to this circumstance, the ESR acquires two important aspects.

(A) First, the BSOC mixes the spin and orbital electron states causing spin transitions to be allowed not only under the action of the magnetic component of the incident radiation but also under the action of the electric component. This idea has been put forward long ago;[15] subsequently it was experimentally realized for donor-bound electrons in bulk $Cd_{1-x}Mn_xSe$ crystals of wurtzite symmetry.[16] Nowadays it has attracted a new interest from the field of spintronics where it has became a basis for developing various mechanisms of the control of the charge-carrier spin via the control of its orbital motion.[17] The present paper represents an attempt to provide a general description of the resonant absorption due to both the magnetic and electric components of the microwave field. Within the scope of macroscopical electrodynamics, the absorption of an electromagnetic wave due to magnetodipole transitions is defined by the imaginary part of the spin susceptibility multiplied by the square of the magnetic field amplitude of the wave. We show below that electrodipole transitions, which are allowed by the presence of the BSOC, lead to a possibility to excite the ESR by the electrical component of the radiation, i.e., to a contribution to the absorption described by the real part of the conductivity multiplied by the square of the electric field amplitude. Moreover, we show that the electric field can excite the ESR more effectively that the magnetic field. There also appear magnetoelectric contributions to the absorption which are bilinear in amplitudes of magnetic and electric fields. These contributions are defined by some macroscopic kinetic coefficients that characterize a conducting media with destroyed mirror symmetry in addition to the magnetic susceptibility and electrical conductivity.

Previous considerations of the electric-dipole-induced ESR (Ref. 7 and 15) were focused on clean systems. Accordingly, they reduced to calculations of spin-flip matrix elements responsible for the excitation of the resonance by the electric (and magnetic) component of the incident radiation relying on the quantum mechanic perturbation theory. The decay of the ESR was taken into account simply by introducing a phenomenological relaxation time into final equations. As opposed to that, the resonance in moderately impure systems in the regime of frequent collisions with impurities is the subject of the present paper. Following the Bloch approach,[18] we consider the resonance as a motion of the ensemble-averaged spin (and current) density under the influence of the external constant magnetic field, the impurity scattering, the BSOC, and the oscillating microwave field. Thus, the calculation of the resonant responses of a system is now a quantum kinetic rather than quantum mechanic problem.

(B) Another one aspect which the BSOC adds to the ESR is a possibility for the orbital motion to influence the spin dynamics. One example of such an influence discovered in Ref. 19 is an "orbital" mechanism of the spin relaxation—the scattering on a scalar, *spin-independent* potential should give rise to the decay of the spin magnetization. Indeed, for an electron with the momentum $\mathbf{p}$, the term, Eq. (1), can be considered as the Zeeman energy of the electron in a fictitious magnetic field $\mathbf{B}_f(\mathbf{p})=\alpha(\mathbf{p}\times\mathbf{c})/g\mu_B$, where g is the electron g factor and μ_B is the Bohr magneton. Hence the spin of the electron precesses about $\mathbf{B}_f(\mathbf{p})$. If, as a result of scattering, the electron goes from a state with the momentum $\mathbf{p}$ into a state with the momentum $\mathbf{p}'$, its spin will appear under the action of the field $\mathbf{B}_f(\mathbf{p}')$ and will have to precess about the new direction. In this way, a stochastic process of impurity scattering inducing the random jumps of the electron on the Fermi surface gives rise to a corresponding stochastic process of the fictitious magnetic field reorientation that leads to a stochastic disturbance of the phase of the spin precession. The randomization of the spin phase results in a finite time of "forgetting" by the electron of its initial spin orientation that reveals itself through the spin magnetization decay. In the paper,[19] only the decay of the spin polarization at zero magnetic field was considered within a quasiclassical approach and the relaxation time $\tau_{so}\simeq\tau_0\eta_0^{-2}$, where τ_0 is the zero-field electron-scattering time and $\eta_0=2\alpha p_F\tau_0$, was obtained for the case of relatively weak BSOC $\eta_0\ll 1$. The transverse relaxation time T_2 through this mechanism first estimated in Ref. 11 also appeared to be proportional to τ_{so}. A rigorous quantum kinetic theory of the ESR developed later[20] has confirmed the result.

All theoretical results of Refs. 11 and 20 were obtained by neglecting the effect of an external magnetic field on the orbital dynamics. Experiments on the ESR, however, suppose a finite external magnetic field $B_{(0)}$ and the effect of the field on the orbital motion may be much more pronounced than on the spin motion. Indeed, because of small effective mass, the cyclotron frequency ω_c of many of semiconductor materials greatly exceeds the Larmour frequency ω_s. Due to this fact, the cyclotron frequency can become equal to or exceed the electron-scattering rate τ_0^{-1} even at moderately strong magnetic fields what makes it necessary to take the cyclotron motion or, in the quantum mechanics language, the orbit quantization into account. From a general viewpoint, one may expect that the cyclotron motion influences the ESR in systems with the BSOC in two ways. First, the cyclotron character of electron motion alters the stochastic process of the electron diffusion on the Fermi surface induced by impurity scattering thereby affecting the described above orbital mechanism of spin relaxation. In addition, both the cyclotron and Larmour motions of the electron density are somewhat circularlike. Therefore, one may suppose that, because of the BSOC, the cyclotron motion can influence the spin precession. Thus both aspects of the ESR, dissipative and dynamical ones, can be subject to the cyclotron motion.

The quantum approach to any kinetic problem under Landau quantization conditions is known to encounters some difficulties. First, the necessity of dealing with a discrete energy spectrum suggests the application of more refined mathematical means than at the absence of the quantization. There is also an additional difficulty specific to systems with the BSOC. The fact is that usually at a microscopic description of the ESR, one needs to know the explicit form of quantum states transitions between which (due to the electromagnetic interaction with the incident radiation as well as impurity and/or phonon scattering) form the resonant absorption. However, in the case under study, the one-particle Hamiltonian can be explicitly solved only at the external magnetic field perpendicular to the plane of the system.[15] But even in that case, a complicated form of the eigenfunctions strongly impedes real calculations. Thus a search for a more adequate formalism is an actual problem.

In the present paper, we circumvent difficulties mentioned above and propose a general method which can be applied to kinetic problems in conductors with the BSOC and which does not require the explicit form of exact one-particle states—their energies and eigenfunctions. For sufficiently dirty semiconductors with the weak BSOC, the method allows one to include in a systematic way effect of the orbital quantization into physics of the ESR in particularly and into spin-dependent electron kinetics in general. Basically, the applied approach reduces to a "kinetic" variant of perturbation theory when some quantities of dynamical nature (the Zeeman energy $\omega_s = g\mu_B B_{(0)}$ and the energy of the spin-orbit splitting $\Delta_{so} = 2\alpha p_F$) are considered to be small in comparison with a quantity of dissipative origin, namely, with the impurity scattering rate τ_0^{-1}. Our principal observation is that main constituents of the quantum kinetic theory (vertexes and kernels of equations controlling spin-dependent kinetic processes) can be calculated simply by treating the Zeeman interaction and H_{so} as small perturbations. The realm of applicability of the method is the "BSOC weak limit," where the SO splitting is weak in the sense that $\Delta_{so} \ll \max(\tau_0^{-1}, \omega_c)$. Thus the approach is applicable at any value of the cyclotron energy from very small $\omega_c \tau_0 \ll 1$ to moderately big $\omega_c \tau_0 \gg 1$, $\omega_c \ll \epsilon_F$, and at any direction of the static magnetic field from perpendicular to parallel to the plane of a 2D system. We show that in addition to a modification of the longitudinal and transversal relaxation times the cyclotron motion influence the frequency of the ESR resulting in a shift of the resonant frequency as compared with the Larmour frequency ω_s. The shift depends on the electron mean-free path, which shows its kinetic origin, and can be positive or negative in a general case. In the particular case of zero g factor, when the Larmour frequency vanishes, the spin precession is maintained solely by the cyclotron motion. It should be emphasized that the analytical form of results presented below appears to be possible due to the employment of two technical means: (i) the method of generating functions proposed in Ref. 21 that greatly simplifies calculations of integrals of the Landau-problem eigenfunctions and (ii) a very convenient method of evaluating sums over Landau levels by means of the contour integration developed in Ref. 22.

There are several attempts in the literature to calculate effects of the change in the orbital dynamics due to the external magnetic field on the spin relaxation and resonance in systems under discussion. On a semiclassical level, effects of the cyclotron motion on the spin dynamics were first addressed in Ref. 23 where a dependence of the longitudinal relaxation time T_1 on the value of an applied magnetic field was calculated. A bibliography of works devoted to the spin relaxation can be found in Ref. 24. An entirely quantum approach for the calculation of both the longitudinal and transverse relaxation times was proposed in a paper.[25] It was restricted to perpendicular fields being entirely relied on the fact that only in that case energies and eigenfunctions of the one-particle Hamiltonian can be represented in an explicit form.[15] Although initial equations of Ref. 25 are valid at any values of material parameters characterizing the system, the awkwardness of some mathematical expressions due to the cumbersome form of the eigenfunctions eventually forced those authors to admit limitations in order to obtain final results in a visible form. The first limitation is $\alpha p_F \tau_0 \ll 1$, which means that the BSOC is relatively weak while the second one is $\omega_s \tau_0 \ll 1$, which assumes that the magnetic field is not very strong or the semiconductor is not very clean. An extension to the case of a tilted magnetic field has been performed in Ref. 26 where a dependence of the relaxation time on angle between the magnetic field and the plane of electron motion as well as the space dispersion of the resonance was calculated. A departure of the spin precession axis from the direction defined by magnetic field under the action of the cyclotron motion was also noticed. In the case of perpendicular magnetic fields and if one omits the space dispersion, results of Ref. 20 mainly agree with those of Ref. 25. However, the approach of Ref. 20 suffers from an undesirable restriction. Namely, the inequality $\Delta_{so} \ll \omega_c$, where $\omega_c = eB_\perp^{(0)}/mc$ is the cyclotron frequency corresponding to the perpendicular component of the field, that was adopted on a stage of that derivation, excludes the limit of magnetic fields parallel to the electron motion plane, when $\omega_c \tau_0 \to 0$ and the Landau quantization becomes ineffective. However, in spite of this fact, the final expression for the resonant spin susceptibility appeared to be well behaved at any directions of the external field coinciding in the parallel field limit with a corresponding expression for the susceptibility obtained earlier[20] by disregarding diamagnetism from the outset. This circumstance tempts us to think that the restriction $\Delta_{so} \ll \omega_c$ at small magnetic fields is not motivated by an essential physics but should be rather a drawback of the theoretical method applied. (Note that Ref. 25 is also not free from an analogous restriction.) The ESR in tilted magnetic fields was also considered in Ref. 27 by means of a semiclassical kinetic equation. However, the correctness of a procedure of exclusion of orbital degrees of freedom to get a balance-type equation for the magnetization, which was not presented there, is not clear. So it is difficult to estimate the validity of the approach applied and the accuracy of results obtained. The effect of the cyclotron motion on frequency and axis of the spin precession was missed in Ref. 27. An attempt to analyze some features of the ESR absorption within a semiclassical approach was also made in Ref. 28. It should be noticed that previously published works considered only the resonant response of the spin density to the Zeeman interaction with the magnetic field, i.e., the spin susceptibility, and to ac electric field.[29] To the best of the author's knowledge, a microscopic treatment of the resonant contribution to the conductivity of the electron gas was never proposed.

The paper is organized as follows. In Secs. II–IV main results of the formal analysis are presented, technical details of derivation being given in Appendices A and D. In Sec. II we introduce the model we aim to investigate. We start with a formulation of constitutive relations that express the densities of electric current **J** and spin magnetization **M** as functions of the electric and magnetic fields **E** and **B**, modified for a conducting medium of polar symmetry. We show that a microscopic cause of additional terms in the constitutive relations is the presence of cross spin-current correlations which emerges due to the BSOC. Further, as a prelude, we present a short evaluation of the additional terms in the simplest case of the absence of an external constant magnetic field. This calculation allows us to formulate our method most clearly, disregarding complexities connected with the Landau quantization. In the end of Sec. II we give an expression for the microwave field absorption which follows from the modified constitutive relations. It is relevant to a general case when the system is subject to both the magnetic and electric components of the field. In Sec. III, we demonstrate the method in a full generality. Although the central object of this paper is the electrical conductivity, we consider first the spin susceptibility tensor, $\chi_{ij}(\omega)$, which determines the absorption of the microwave radiation when a 2D structure is placed in a node of electric component of the microwave field. The fact is that the operator of current, the correlation function of which defines the conductivity according to rules of quantum statistical mechanics,[30] is a function of coordinates, whereas the spin operator is not. As will be seen, because of this circumstance, the evaluation of $\chi_{ij}(\omega)$ appears to be more simple than $\sigma_{ij}(\omega)$. We show that results of Ref. 26 are really valid at any relation between Δ_{so} and ω_c. In Sec. IV, we apply the method to a calculation of the cross spin-current-correlation tensors $\gamma_{ij}(\omega)$ and $\theta_{ij}(\omega)$. In Sec. V, we calculate the electric conductivity tensor, $\sigma_{ij}(\omega)$ which determines the absorption of the microwave radiation when the 2D structure is placed in an antinode of electric component of the microwave field. Finally, in Sec. VI, we summarize our results and give an outlook for possible further investigations.

II. MODEL AND FORMULATION

A. Model Hamiltonian

The physical model we are using is based on the following premises. We assume that the Coulomb energy is much smaller than the Fermi energy ϵ_F so that many-body effects do not play an appreciable role and may be omitted. Thus we consider the 2D degenerate system of electrons of charge $-e$ and spin $\frac{1}{2}$ subject to a moderately strong external magnetic field $\mathbf{B}_{(0)}$, $\omega_c \ll \epsilon_F$. Then the matter Hamiltonian has the form

$$H = H_0 + H_{so} + H_Z + H_{imp},$$

$$H_0 = \frac{1}{2m}\left(\mathbf{p} + \frac{e}{c}\mathbf{A}_{(0)}\right)^2, \quad H_{so} = \alpha\left(\mathbf{p} + \frac{e}{c}\mathbf{A}_{(0)}\right) \times \mathbf{c} \cdot \boldsymbol{\sigma},$$

$$H_Z = -\boldsymbol{\mu} \cdot \mathbf{B}_{(0)}, \quad H_{imp} = \sum_i U(\mathbf{r} - R_i). \tag{2}$$

Here $\boldsymbol{\mu} = -g\mu_B\boldsymbol{\sigma}/2$ is the spin magnetic-moment operator, $\mathbf{A}_{(0)} = \frac{1}{2}(\mathbf{B}_\perp^{(0)} \times \mathbf{r})$ is the vector potential of the part of the field that is perpendicular to the plane of the system, $\mathbf{B}_\perp^{(0)} = \mathbf{c}(\mathbf{c} \cdot \mathbf{B}_{(0)})$, and the potential of impurities positioned in arbitrary distributed points $\mathbf{R}_i$ of concentration n_{imp} is considered to be short ranged: $U(\mathbf{r} - \mathbf{R}_i) = U\delta(\mathbf{r} - \mathbf{R}_i)$ [then according to Ref. 30 the elastic lifetime τ_0 is given by $\tau_0^{-1} = mn_{imp}U^2$].

There are several dimensionless parameters determining the behavior of the system. As it was mentioned above, the external magnetic field appears in the theory through the parameters $\omega_s\tau_0$ and $\omega_c\tau_0$, where $\omega_s = g\mu_B B^{(0)}$ and $\omega_c = eB_\perp^{(0)}/mc$ are the Larmour and cyclotron frequencies. The first parameter is treated below as small. But because the cyclotron frequency in many semiconductors may exceed very much the Larmour frequency, the parameter $\omega_c\tau_0$ can be large. Spin-orbit constant α also appears in the problem in two ways. The parameter $\delta = \alpha p_F/\epsilon_F$, where $\epsilon_F = p_F^2/2m$ is the Fermi energy, having a pure quantum-mechanical nature is treated as being very small so that all powers of δ in equations in excess of the first can be ignored. Another parameter $\eta_0 = 2\alpha p_F\tau_0$, which controls the kinetics of spin-flip process by impurity scattering, is much greater $\eta_0/\delta = 2\epsilon_F\tau_0 \gg 1$. In this paper we adopt $\eta_0 \ll 1$, i.e.,

$$\delta \ll \eta_0 \ll 1. \tag{3}$$

Note that the inequality $\omega_s\tau_0 \ll 1$ does not prevent the ESR to be sharp because its width is determined by the spin relaxation time $\tau_{so} \sim \tau_0\eta_0^{-2}$, which is much longer than the mean-free time τ_0. So the sharp-resonance condition (if one neglects effects of the cyclotron motion) is

$$\eta_0^2 \ll \omega_s\tau_0. \tag{4}$$

B. Constitutive relations

It is natural to expect that the parity violation with respect to reflection in the plane of the 2D electron layer should influence macroscopical properties of the substance. The magnetoelectric effect (MEE)—the occurrence of a spin polarization of the current carriers induced by the electric current, predicted in Ref. 31 and experimentally observed in Ref. 32, is an example of such an influence. It can be expressed through the relation

$$\mathbf{S} = d\left(\mathbf{c} \times \frac{\mathbf{J}}{ev_F}\right), \tag{5}$$

where $\mathbf{S}$ is the spin-polarization density, $\mathbf{J}$ is the current density, v_F is the Fermi velocity, and d is a constant proportional to the spin-orbit constant α. Such a relation would be forbidden in a center-symmetric system because spin is a parity-even quantity whereas current is parity odd. On the microscopic level, the property that ensures the MEE is spin-velocity correlations, which are inherent in systems with the BSOC. In a more general case of a time-dependent electromagnetic field (sufficiently slow and weak), a result of the presence of the spin-velocity correlations is a modified form of constitutive relations

$$\mathbf{M} = \hat{\chi}\mathbf{B} + \hat{\gamma}\mathbf{E}, \tag{6}$$

$$\mathbf{J} = \hat{\sigma}\mathbf{E} - \hat{\theta}\dot{\mathbf{B}}, \tag{7}$$

where $\mathbf{M}$ is the spin magnetization density, $\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic components of the field and a point over $\mathbf{B}$ denotes the time derivative.[33] First terms in right-hand sides of these equations (with the conductivity $\hat{\sigma}$ and the spin susceptibility $\hat{\chi}$) are familiar, whereas the presence of second terms (with kinetic coefficients $\hat{\gamma}$ and $\hat{\theta}$) is one of the hallmarks of electrodynamics of conducting media of polar symmetry. It is seen that right-hand sides of Eqs. (6) and (7) are sums of terms of different parity under space reversal. The second term in Eq. (6) accounts for the MEE in the stationary limit. Strictly speaking, the coefficient $\hat{\chi}$ in Eq.

(6) slightly differs from the standard susceptibility since it relates the magnetic moment $\mathbf{M}$ with the magnetic induction $\mathbf{B}$ rather than the magnetic field $\mathbf{H}$. But because $\hat{\chi} \ll 1$, we will neglect the difference. It has to be emphasized that the form of Eqs. (6) and (7) is somewhat conditional. It implies that the static part of the magnetic field, $\mathbf{B}_{(0)}$, and consequently the equilibrium part of the spin magnetization, $\mathbf{M}_{(0)}$, are excluded from $\mathbf{B}$ and $\mathbf{M}$, respectively. The static magnetic field, which does not need to be small, has been included in the matter Hamiltonian (2); accordingly, its influence on the material kinetic coefficients is assumed.

A connection between the spin-velocity correlations and the additional terms can be shown in the following way. For the sake of simplicity, we consider here the case without the external magnetic field; a general case will be considered in subsequent sections. In the linear-response regime, $\mathbf{M}$ and $\mathbf{J}$ can be expanded in the electromagnetic interaction

$$H_{ef} = -\frac{1}{c}\int d^2r \mathbf{j}(\mathbf{r}) \cdot \mathbf{A}(\mathbf{r},t). \tag{8}$$

Since, due to the BSOC, the velocity operator

$$\mathbf{v}(\mathbf{p}) = i[H,\mathbf{r}] = \frac{\mathbf{p}}{m} + (\mathbf{c} \times \sigma) \tag{9}$$

as well as the usual scalar part $\mathbf{v}^{(sc)}$ also has a spin component $\mathbf{v}^{(sp)}$, the current operator in the second quantization representation has the form

$$\mathbf{j} = \mathbf{j}_{kin} + \mathbf{j}_{dia} + \mathbf{j}_{par},$$

$$\mathbf{j}_{kin} = -\frac{e}{m}\left(\psi^+_\beta \frac{\nabla}{2i}\psi_\beta - \psi_\beta \frac{\nabla}{2i}\psi^+_\beta\right) - e\alpha\psi^+_\beta(\mathbf{c} \times \sigma)_{\beta\gamma}\psi_\gamma,$$

$$\mathbf{j}_{dia} = -\frac{e^2}{mc}\mathbf{A}\psi^+_\beta\psi_\beta, \quad \mathbf{j}_{par} = c\,\nabla \times (\psi^+_\beta \boldsymbol{\mu}_{\beta\gamma}\psi_\gamma). \tag{10}$$

Accordingly, the interaction Hamiltonian can be rewritten as

$$H_{ef} = H^{(1)}_{ef} + H^{(2)}_{ef},$$

$$H^{(1)}_{ef} = -\int d^2r \boldsymbol{\mu}(\mathbf{r}) \cdot \mathbf{B}(\mathbf{r},t),$$

$$H^{(2)}_{ef} = -\frac{1}{c}\int d^2r[\mathbf{j}_{kin}(\mathbf{r}) + \mathbf{j}_{dia}(\mathbf{r})] \cdot \mathbf{A}(\mathbf{r},t), \tag{11}$$

where $\boldsymbol{\mu}(\mathbf{r}) = -(g\mu_B/2)\psi^+_\beta(\mathbf{r})\boldsymbol{\sigma}_{\beta\gamma}\psi_\gamma(\mathbf{r})$ and $\mathbf{B} = \nabla \times \mathbf{A}$. First terms in Eqs. (6) and (7) are usual responses of the spin magnetization on $H^{(1)}_{ef}$ and the current on $H^{(2)}_{ef}$. Let us show that additional terms arise as a cross responses of the spin magnetization on $H^{(2)}_{ef}$ and the current on $H^{(1)}_{ef}$. According to rules of statistical physics,[30] we have for the second term in Eq. (6)

$$M^{(2)}_i(\omega_n) = -\left(\frac{g}{2}\mu_B\right)T\sum_{\epsilon_l}\int_{\mathbf{p}} \mathrm{Tr}\{\sigma_i \times G(\epsilon_l + \omega_n,\mathbf{p})v_j(\mathbf{p})G(\epsilon_l,\mathbf{p})\}\frac{e}{c}A_j(\omega_n), \tag{12}$$

where $\epsilon_l = i\pi T(2l+1)$, $\omega_n = i\pi T2n$ are fermion and boson Matsubara frequencies,[34] $\mathbf{v}(\mathbf{p})$ is given by Eq. (9), $G(\epsilon_l,\mathbf{p})$ is the Green's function corresponding to Hamiltonian (2), $\int_{\mathbf{p}} = \int \frac{d^2p}{(2\pi)^2}$, and effects of impurities are temporary omitted. Only the contribution of $\mathbf{j}_{kin}$ is retained; an analogous contribution of $\mathbf{j}_{dia}$ can be shown to be zero. In the Feynman-diagram language, the right-hand side of Eq. (12) is the fermion loop with two vertexes one of which (the left response vertex) contains the operator σ_i and another (the right cause vertex) is the velocity operator v_j. The term "spin-velocity correlations" adopted in this paper means a nonzero value of diagrams of such a type. As is known,[30] in order to get the Fourier-component $M^{(2)}_i(\omega)$ at real frequencies an analytical continuation from the discrete frequencies ω_n to the real axis from above $\omega + 0^+$, where $\omega \in (-\infty,\infty)$ is the real variable, should be performed. Referring the reader to Ref. 30 for details of the continuation we give the final result. The loop becomes three diagrams: one retarder diagram that involves the product $G^R(\epsilon+\omega,\mathbf{p})v_j(\mathbf{p})G^R(\epsilon,\mathbf{p})$ multiplied by $\tanh(\frac{\epsilon}{2T})$, one advanced diagram that involves the product $G^A(\epsilon+\omega,\mathbf{p})v_j(\mathbf{p})G^A(\epsilon,\mathbf{p})$ multiplied by $-\tanh(\frac{\epsilon+\omega}{2T})$, and one diagram which will be called kinetic. The latter involves the product $G^R(\epsilon+\omega,\mathbf{p})v_j(\mathbf{p})G^A(\epsilon,\mathbf{p})$ multiplied by the function $F(\epsilon,\omega) = [\tanh\frac{\epsilon+\omega}{2T} - \frac{\epsilon}{2T}]$. Since two quasiparticle poles in the product $G^R(\epsilon+\omega,\mathbf{p})v_j(\mathbf{p})G^R(\epsilon,\mathbf{p})$ [and also in $G^A(\epsilon+\omega,\mathbf{p})v_j(\mathbf{p})G^A(\epsilon,\mathbf{p})$] lie on the same side of the real axis, the effective range of the momentum integration in advanced and retarded diagrams is of order p_F (Ref. 35) so that one may ignore the frequency ω of the electromagnetic field and the presence of impurities which is accurate up to perturbations on the order of ω/ϵ_F and $(\epsilon_F\tau_0)^{-1}$. The contributions of these diagrams become proportional to

$$\int_{\mathbf{p}} \mathrm{Tr}\{\sigma_i G^{R(A)}(\epsilon,\mathbf{p})v_j(\mathbf{p})G^{R(A)}(\epsilon,\mathbf{p})\} \tag{13}$$

and hence is zero since the integrand, due to the Ward identity $G^{R(A)}(\epsilon,\mathbf{p})v_j(\mathbf{p})G^{R(A)}(\epsilon,\mathbf{p}) = \frac{\partial}{\partial p_j}G^{R(A)}(\epsilon,\mathbf{p})$, is the full derivative. In this way we come to

$$M^{(2)}_i(\omega) = -\left(\frac{g}{2}\mu_B\right)\int \frac{d\epsilon}{2\pi i}\omega N'(\epsilon,\omega)\int_{\mathbf{p}} \mathrm{Tr}\{\sigma_i G^R(\epsilon+\omega,\mathbf{p}) \times v_j(\mathbf{p})G^A(\epsilon,\mathbf{p})\}\frac{e}{c}A_j(\omega), \tag{14}$$

where $N'(\epsilon,\omega) = \frac{1}{\omega}[f_F(\epsilon) - f_F(\epsilon+\omega)]$, $f_F(\epsilon) = [\exp(\frac{\epsilon-\mu}{T}) + 1]^{-1}$ is the Fermi distribution function. Thus, in the zero-temperature limit, we get for the kinetic coefficient $\hat{\gamma}$ of Eq. (6)

$$\gamma_{ij}(\omega) = \left(\frac{eg\mu_B}{4\pi}\right)\int_{\mathbf{p}} \mathrm{Tr}\{\sigma_i G^R(\epsilon_F+\omega,\mathbf{p})v_j(\mathbf{p})G^A(\epsilon_F,\mathbf{p})\}. \tag{15}$$

An analogous consideration of the response of the electric current to $H_{ef}^{(1)}$ leads to

$$\theta_{ij}(\omega) = \left(\frac{eg\mu_B}{4\pi}\right)\int_{\mathbf{p}} \mathrm{Tr}\{v_i(\mathbf{p})G^R(\epsilon_F+\omega,\mathbf{p})\sigma_j G^A(\epsilon_F,\mathbf{p})\}. \tag{16}$$

Equations (15) and (16) express the kinetic coefficients $\hat{\gamma}$ and $\hat{\theta}$ through the spin-velocity correlations like the standard equations for the conductivity[30]

$$\sigma_{ij}(\omega) = \frac{e^2}{2\pi}\int_{\mathbf{p}} \mathrm{Tr}\{v_i(\mathbf{p})G^R(\epsilon_F+\omega,\mathbf{p})v_j(\mathbf{p})G^A(\epsilon_F,\mathbf{p})\} \tag{17}$$

and the dynamical part of the spin susceptibility

$$\chi_{ij}^{dyn}(\omega) = \frac{i\omega}{2\pi}\left(\frac{g}{2}\mu_B\right)^2\int_{\mathbf{p}} \mathrm{Tr}\{\sigma_i G^R(\epsilon_F+\omega,\mathbf{p})\sigma_j G^A(\epsilon_F,\mathbf{p})\} \tag{18}$$

express these quantities through the dynamical velocity-velocity and spin-spin correlations. An account of impurity scattering is known to lead to the appearance of a decay of the Green's function and to the impurity renormalization of one of two vertexes. For $\chi_{ij}^{(dyn)}(\omega)$, it is equivalent to the change $\sigma_i \to \Sigma_i$, where Σ_i is the solution of the ladder-type vertex equation

$$\Sigma^i_{\alpha\beta}(\omega) = \sigma^i_{\alpha\beta} + \frac{1}{m\tau_0}\int_{\mathbf{p}} G^A_{\alpha\gamma}(\epsilon_F,\mathbf{p})\Sigma^i_{\gamma\rho}(\omega)G^R_{\rho\beta}(\epsilon_F+\omega,\mathbf{p}). \tag{19}$$

In the case of $\hat{\gamma}(\omega)$ and $\hat{\theta}(\omega)$, one may also consider that the spin vertex is to be impurity renormalized. In the case of the conductivity, one of bar velocity vertexes, for example, the left one $v_i(\mathbf{p})$, should be renormalized which means the substitution $v_i(\mathbf{p}) \to V^i$, where the vertex V^i is defined by the equation

$$V^i_{\alpha\beta}(\omega,\mathbf{p}) = v^i_{\alpha\beta}(\mathbf{p}) + \frac{1}{m\tau}\int_{\mathbf{k}} G^A_{\alpha\kappa}(\epsilon_F,\mathbf{k}) \times V^i_{\kappa\delta}(\omega,\mathbf{k})G^R_{\delta\beta}(\epsilon_F+\omega,\mathbf{k}). \tag{20}$$

C. Outline of the method

In this section, we present shortly the logic of our approach to spin-dependent kinetic problems by considering the cross-response coefficient $\hat{\theta}$; details of calculations are given in Appendix A. As it is seen from Eqs. (16) and (19), the basic constituents of the theory after the averaging over impurity positions is performed are the kernel of Eq. (19)

$$(\delta\kappa|t(\omega)|\alpha\beta) \equiv t(\omega)_{\delta\beta,\alpha\kappa} \overset{def}{=} \frac{1}{m\tau_0}\int_{\mathbf{p}} G^R_{\delta\beta}(\epsilon_F+\omega,\mathbf{p})G^A_{\alpha\kappa}(\epsilon_F,\mathbf{p}), \tag{21}$$

which allows one to rewrite Eq. (19) as

$$\mathbf{\Sigma}_{\delta\kappa}(\omega) = \boldsymbol{\sigma}_{\delta\kappa} + (\delta\kappa|t(\omega)|\alpha\beta)\mathbf{\Sigma}_{\beta\alpha}(\omega), \tag{22}$$

and the velocity-vertex $\hat{\mathbf{v}}^{(1)}$

$$\mathbf{v}^{(1)}_{\kappa\beta}(\omega) = \frac{1}{m\tau_0}\int_{\mathbf{p}} [G^A(\epsilon_F,\mathbf{p})\mathbf{v}(\mathbf{p})G^R(\epsilon_F+\omega,\mathbf{p})]_{\kappa\beta}, \tag{23}$$

which has the sense of the first impurity correction to the bare velocity operator. The factor $1/m\tau_0$ is introduced to Eq. (21) to make the kernel dimensionless. The kinetic coefficient $\hat{\theta}$ is expressed through the solution of Eq. (22) and the velocity-vertex $\mathbf{v}^{(1)}$ as

$$\theta_{ij}(\omega) = m\tau_0\left(\frac{eg\mu_B}{4\pi}\right)\mathrm{Tr}\{v^i_{(1)}\Sigma^j(\omega)\}. \tag{24}$$

The electron Green's function of Hamiltonian (2) at $\mathbf{B}_{(0)}=0$ has the form (see, e.g., Refs. 20 and 31)

$$G_{\alpha\beta}(\epsilon_n,\mathbf{p}) = \sum_{\nu=\pm}\Pi^{(\nu)}_{\alpha\beta}(\mathbf{p})G_{(\nu)}(\epsilon_n,p), \tag{25}$$

$$G_{(\nu)}(\epsilon_n,p) = [\tilde{\epsilon}_n - \xi_{(\nu)}]^{-1}, \tag{26}$$

$$\Pi^{(\pm)}_{\alpha\beta}(\mathbf{p}) = \frac{1}{2}[\delta_{\alpha\beta} \pm (\hat{\mathbf{p}}\times\mathbf{c})\cdot\boldsymbol{\sigma}_{\alpha\beta}], \tag{27}$$

where $\xi_{(\pm)}(p)=\epsilon_{(\pm)}(p)-\mu$, $\epsilon_{(\pm)}(p)=p^2/2m \pm \alpha p$, and $\tilde{\epsilon}_n = \epsilon_n[1+(2\tau_0|\epsilon_n|)^{-1}]$. The simplicity of this form allows one to evaluate $\hat{\theta}$ without difficulty.[20] We, however, will use another method which is adequate in a more general case when the system is subject to an external magnetic field. Namely, we will expand the quantities [Eqs. (21) and (23)] in a power series about small parameters $\omega\tau$, η, and δ. [The presence of the external magnetic field will be seen below to add another one small parameter $(\omega_s\tau)$.] The expansion can be obtained simply by means of the expansion of the Green's function $G_{\alpha\beta}(\epsilon_n,\mathbf{p})$,

$$G(\epsilon_n,\mathbf{p}) = G^{(0)}(\epsilon_n,\mathbf{p}) + G^{(0)}(\epsilon_n,\mathbf{p})H_{so}(\mathbf{p})G^{(0)}(\epsilon_n,\mathbf{p}) + G^{(0)}(\epsilon_n,\mathbf{p})H_{so}(\mathbf{p})G^{(0)}(\epsilon_n,\mathbf{p})H_{so}(\mathbf{p})G^{(0)}(\epsilon_n,\mathbf{p}) + \cdots, \tag{28}$$

where $G^{(0)}_{\alpha\beta}(\epsilon_n,\mathbf{p})=\delta_{\alpha\beta}[\tilde{\epsilon}_n-\epsilon(p)-\mu]^{-1}$ is the Green's function of the system with removed spin-orbit coupling. By substituting Eq. (28) into Eqs. (21) and (23), one obtains expansions of the kernel $t(\omega)$ and the vertex $\hat{\mathbf{v}}^{(1)}(\omega)$. The evaluation of terms of the expansions can be carried out in a standard manner. In the integrals over the momentum space, one should change the Cartesian coordinates for polar coordinates. Then angular integration gives rise to a combination of Pauli matrices and the remaining radial integrals can be elementary performed with the help of the theory of residues.

First consider the kernel $\hat{t}(\omega)$. Because of the integration over $\mathbf{p}$ direction, only terms with even numbers of H_{so} contribute. To an accuracy of α^2 terms, we have

$$\hat{t}(\omega) \simeq \hat{t}_{(0)}(\omega) + \hat{t}_{(2)}, \tag{29}$$

where

$$(\delta\kappa|t_{(0)}|\alpha\beta) = \frac{1}{m\tau_0}\int_{\mathbf{p}} G^{R(0)}_{\delta\beta}G^{A(0)}_{\alpha\kappa} \tag{30}$$

and

$$\hat{t}_{(2)} = \hat{p}_{(11)} + \hat{p}_{(02)} + \hat{p}_{(20)},$$

$$(\delta\kappa|p_{(11)}|\alpha\beta) = \frac{1}{m\tau_0}\int_{\mathbf{p}} G^{R(1)}_{\delta\beta}G^{A(1)}_{\alpha\kappa},$$

$$(\delta\kappa|p_{(02)}|\alpha\beta) = \frac{1}{m\tau_0}\int_{\mathbf{p}} G^{R(0)}_{\delta\beta}G^{A(2)}_{\alpha\kappa},$$

$$(\delta\kappa|p_{(20)}|\alpha\beta) = \frac{1}{m\tau_0}\int_{\mathbf{p}} G^{R(2)}_{\delta\beta}G^{A(0)}_{\alpha\kappa} \tag{31}$$

Here

$$G^{(1)} = G^{R(0)}H_{so}G^{R(0)}, \quad G^{(2)} = G^{R(0)}H_{so}G^{R(0)}H_{so}G^{R(0)}. \tag{32}$$

In Eqs. (29)–(31) and all below, the energy and momentum arguments $(\epsilon_F+\omega,\mathbf{p})$ of retarded Green's functions and $(\epsilon_F,\mathbf{p})$ of advanced Green's functions are omitted for brevity. A calculation yields (see Appendix A)

$$(\delta\kappa|t_{(0)}(\omega)|\alpha\beta) \cong (1+i\omega\tau_0)\delta_{\delta\beta}\delta_{\alpha\kappa},$$

$$(\delta\kappa|p_{(11)}|\alpha\beta) \cong \frac{\eta_0^2}{2}(\mathbf{c}\times\boldsymbol{\sigma})_{\delta\beta}(\mathbf{c}\times\boldsymbol{\sigma})_{\alpha\kappa},$$

$$(\delta\kappa|p_{(02)}|\alpha\beta) = (\delta\kappa|p_{(20)}|\alpha\beta) \cong -\frac{\eta_0^2}{4}\delta_{\delta\beta}\delta_{\alpha\kappa}. \tag{33}$$

Note that the right-hand sides of Eq. (33) include the direct products of a matrix which depends on the indices $(\delta\beta)$ and a matrix which depends on the indices $(\alpha\kappa)$. It is much convenient, however, to deal with the direct products of matrices one of which depends on the indices $(\delta\kappa)$ and another one on the indices $(\alpha\beta)$. In the Feynman-diagram language, it means to split up the four spin indices into a pair of indices by means of which the kernel is connected with other part of a ladder diagram coming from the left and a pair of other indices through which the kernel is connected with a part of the ladder diagram coming from the right. An advantage of the representation obtained in such a way is that it allows one to readily reduce the spin-matrix equations for the spin and velocity vertexes to systems of scalar equations. The desired rearrangement of the spin indices is possible owing to Fierz-type identities for the direct products of the Pauli matrices (see Appendix C). By means of these identities $\hat{t}(\omega)$ can be written in the form

$$(\delta\kappa|t(\omega)|\alpha\beta) \cong \frac{1}{2}(1+i\omega\tau_0)\delta_{\delta\kappa}\delta_{\alpha\beta} + \frac{1}{2}(1+i\omega\tau_0-\eta_0^2)(\mathbf{c}\cdot\boldsymbol{\sigma})_{\delta\kappa}(\mathbf{c}\cdot\boldsymbol{\sigma})_{\alpha\beta} + \frac{1}{2}\left(1+i\omega\tau_0-\frac{\eta_0^2}{2}\right)(\mathbf{c}\times\boldsymbol{\sigma})_{\delta\kappa}(\mathbf{c}\times\boldsymbol{\sigma})_{\alpha\beta}. \tag{34}$$

It easy to verify that the solution of Eq. (22) has the form

$$\Sigma_i(\omega) = \frac{c_i(\mathbf{c}\cdot\boldsymbol{\sigma})}{-i\omega\tau_0+\eta_0^2} + \frac{\sigma_i - c_i(\mathbf{c}\cdot\boldsymbol{\sigma})}{-i\omega\tau_0+\frac{1}{2}\eta_0^2}. \tag{35}$$

Now consider $\mathbf{v}^{(1)}(\omega)$. In view of Eqs. (9) and (21),

$$\mathbf{v}^{(1)}(\omega) = \mathbf{v}^{(1,sc)}(\omega) + \mathbf{v}^{(1,sp)}(\omega), \tag{36}$$

where

$$\mathbf{v}^{(1,sc)}_{\alpha\beta}(\omega) = \frac{1}{m\tau_0}\int_{\mathbf{p}}\left[G^A\frac{\mathbf{p}}{m}G^R\right]_{\alpha\beta} \tag{37}$$

and

$$\mathbf{v}^{(1,sp)}_{\alpha\beta}(\omega) = \alpha(\mathbf{c}\times\boldsymbol{\sigma})_{\kappa\delta}(\delta\kappa|t(\omega)|\alpha\beta). \tag{38}$$

Because of the presence of the vector $\mathbf{p}$ under the integral in Eq. (37), only terms with odd numbers of H_{so} contribute. To an accuracy of α^3 terms, we have

$$\mathbf{v}^{(1,sc)}(\omega) = \mathbf{v}^{(sc)}_{(1.1)}(\omega) + \mathbf{v}^{(sc)}_{(1.3)}(\omega), \tag{39}$$

where

$$\mathbf{v}^{(sc)}_{(1.1)\alpha\beta}(\omega) = \frac{1}{m\tau_0}\int_{\mathbf{p}}\frac{\mathbf{p}}{m}[G^{A(1)}G^{R(0)} + G^{A(0)}G^{R(1)}]_{\alpha\beta}, \tag{40}$$

$$\mathbf{v}^{(sc)}_{(1.3)\alpha\beta}(\omega) = \frac{1}{m\tau_0}\int_{\mathbf{p}}\frac{\mathbf{p}}{m}[G^{A(3)}G^{R(0)} + G^{A(2)}G^{R(1)} + G^{A(1)}G^{R(2)} + G^{A(0)}G^{R(3)}]_{\alpha\beta}. \tag{41}$$

A calculation yields (see Appendix A)

$$\mathbf{v}^{(sc)}_{(1.1)}(\omega) \cong -\alpha(\mathbf{c}\times\boldsymbol{\sigma})(1+i\omega\tau_0) \tag{42}$$

and $\mathbf{v}^{(sc)}_{(1.3)}=0$. The corresponding expansion of $\mathbf{v}^{(1,sp)}(\omega)$, with the help of Eq. (34), can be found to be

$$\hat{\mathbf{v}}^{(1,sp)}(\omega) \cong \alpha(\mathbf{c}\times\boldsymbol{\sigma})\left(1+i\omega\tau_0-\frac{\eta_0^2}{2}\right). \tag{43}$$

As a result,

$$\hat{\mathbf{v}}^{(1)}(\omega) \cong -\alpha\frac{\eta_0^2}{2}(\mathbf{c}\times\boldsymbol{\sigma}) \tag{44}$$

does not depend on ω. Substituting Eqs. (35) and (44) into Eq. (24), we get

$$\theta_{ij}(\omega) = \theta(\omega)e_{ijs}c_s, \quad \theta(\omega) = \alpha m\tau_0\left(\frac{eg\mu_B}{2\pi}\right)[1 + i\omega\tau_{so}(0)]^{-1}, \tag{45}$$

where $\tau_{so}(0)^{-1}=\eta_0^2/2\tau_0$. Thus the frequency dispersion of θ, just as χ, becomes essential at frequencies $\omega\sim\tau_{so}^{-1}$ whereas the dispersion of the conductivity appears only at more high frequencies $\omega\sim\tau_0^{-1}$.

An evaluation of the kinetic coefficient $\hat{\gamma}$ is carried out along the same lines. In this case, one should introduce the right velocity vertex

$$\mathbf{v}_{\kappa\beta}^{(1r)}(\omega) = \frac{1}{m\tau_0}\int_{\mathbf{p}}[G^R(\epsilon_F + \omega,\mathbf{p})\mathbf{v}(\mathbf{p})G^A(\epsilon_F,\mathbf{p})]_{\kappa\beta} \tag{46}$$

instead of the left velocity-vertex $\mathbf{v}^{(1l)}$ defined by Eq. (23) and, through the equation

$$\mathbf{\Sigma}_{\gamma\beta}^{(l)}(\omega) = \boldsymbol{\sigma}_{\gamma\beta} + \mathbf{\Sigma}_{\delta\alpha}^{(l)}(\omega)(\alpha\delta|t(\omega)|\gamma\beta), \tag{47}$$

the left renormalized spin vertex $\mathbf{\Sigma}^{(l)}(\omega)$ instead of the right spin vertex $\mathbf{\Sigma}^{(r)}(\omega)$ defined by Eq. (22). In terms of $\mathbf{\Sigma}^{(l)}(\omega)$ and $\mathbf{v}^{(1r)}$, the coefficient $\hat{\gamma}$ can be expressed as

$$\gamma_{ij}(\omega) = m\tau_0\left(\frac{eg\mu_B}{4\pi}\right)\mathrm{Tr}\{\Sigma_i^{(l)}(\omega)v_j^{(1r)}(\omega)\}. \tag{48}$$

It easy to check that $\mathbf{v}^{(1l)}=\mathbf{v}^{(1r)}$ and $\mathbf{\Sigma}^{(l)}=\mathbf{\Sigma}^{(r)}$ so that

$$\gamma_{ji}(\omega) = \theta_{ij}(\omega). \tag{49}$$

Thus, in the absence of an external magnetic field, the constitutive relations take the form

$$\mathbf{M} = \chi\mathbf{B} + \gamma\mathbf{E}\times\mathbf{c}, \tag{50}$$

$$\mathbf{J} = \sigma\mathbf{E} + \gamma\dot{\mathbf{B}}\times\mathbf{c}. \tag{51}$$

In the low-frequency limit $\omega\tau_{so}(0)\ll 1$, Eq. (50) reduces to

$$\mathbf{M} = \left(\frac{eg\mu_B}{2\pi}\right)\alpha m\tau_0(\mathbf{c}\times\mathbf{E}), \tag{52}$$

which coincides with Eq. (5) with $d=\frac{\alpha p_F}{\epsilon_F}$ if one expresses the spin magnetization $\mathbf{M}$ through the spin polarization $\mathbf{S}$ as $\mathbf{M}=\mathbf{S}(g\mu_B/2)$ and the electric field $\mathbf{E}$ through the electric current density as $\mathbf{J}=\sigma_D\mathbf{E}$ (with the Drude conductivity $\sigma_D=e^2p_F^2\tau_0/2\pi m$).

Note that one can interpret the presence of the additional terms in the right-hand sides of Eqs. (50) and (51) in the following way. The emergence of the electric current induced by the electric field $\mathbf{E}$ implies a shift of the Fermi surface on the drift momentum $\mathbf{p}_{dr}\sim e\mathbf{E}\tau$ what means that the fictitious magnetic field $\mathbf{B}_f(\mathbf{p})$ acquires a component $\alpha(\mathbf{p}_{dr}\times\mathbf{c})/g\mu_B\sim\mathbf{E}\times\mathbf{c}$, which is the same for all electrons. The second term in Eq. (50) may be viewed as the spin magnetization due to this field. The existence of its counterpart, i.e., the second term in Eq. (51), may then be expected by taking guidance from the Onsager's symmetry principle.[36]

The calculations just presented show a key point of the approach proposed—one should express kernels of the Bethe-Salpeter-type equations for the exact spin and velocity vertexes (but not solutions of these equations) as a power series in the small parameters. The expressions can be obtained simply by means of the expansion [Eq. (28)] of the full Green's function. Since one-electron line of any ladder diagram that contribute to any of the kinetic coefficients $\hat{\sigma}$, $\hat{\mu}^{(dyn)}$, $\hat{\gamma}$, and $\hat{\theta}$ is retarded whereas the other is advanced, quasiparticle poles multiplied together lie on either side of the real axis. Therefore, the integration over $\xi=\frac{p^2}{2m}-\epsilon_F$ in any element of the ladder diagram is limited to the region on the order of τ_0^{-1} so that the corresponding integrals converge. Thus the Green' function without spin-orbit coupling becomes the main calculational tool of the theory. It is not difficult, therefore, to anticipate that the approach will be efficient as well when the Landau quantization is taken into account. Indeed, the spectral decomposition of the Green' function of the Landau problem is not difficult to use and the substitution of the ξ integration for the summation over the Landau levels should not introduce a principal complication into the method.

D. Absorption coefficient

It is convenient to separate the total current-density operator into the "paramagnetic," $\hat{\mathbf{j}}_{par}$, and "conduction," $\hat{\mathbf{j}}_{cond}=\mathbf{j}_{kin}+\mathbf{j}_{dia}$, parts. Then the power loss of the microwave field due to the Joule heating can be represented as[37]

$$R = \langle\mathbf{J}\cdot\mathbf{E} - \mathbf{M}\cdot\dot{\mathbf{B}}\rangle, \tag{53}$$

where $\mathbf{J}$ is the ensemble-averaged conduction current density, $\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic components of the field, a point over $\mathbf{B}$ denotes the time derivative, and the brackets mean the time averaging. Inserting the constitutive relations (6) and (7) into Eq. (53), we obtain for the case of a monochromatic wave

$$R = \frac{1}{2}[\mathbf{E}_\omega^*\cdot\hat{\sigma}'(\omega)\cdot\mathbf{E}_\omega] + \frac{\omega}{2}[\mathbf{B}_\omega^*\cdot\hat{\chi}''(\omega)\cdot\mathbf{B}_\omega] - \frac{\omega}{2}\mathrm{Im}\{\mathbf{E}_\omega^*\cdot[\hat{\theta}(\omega) + \hat{\gamma}^+(\omega)]\cdot\mathbf{B}_\omega\}, \tag{54}$$

where $\mathbf{E}_\omega$ and $\mathbf{B}_\omega$ are the Fourier amplitudes of the electric and magnetic fields, $\hat{\sigma}'=\mathrm{Re}\,\hat{\sigma}$, $\hat{\chi}''=\mathrm{Im}\,\hat{\chi}$, and the superscript + means the Hermitian conjugation. Here the first, "electric" term, R_E, and the second, "magnetic" term, R_B, are standard. The third, "magnetoelectric" term R_{EB} is a characteristic feature of a conducting media of polar symmetry.

While the magnetoelectric term has been obtained within macroscopic electrodynamics, it can be also interpreted in terms of quantum mechanics. Indeed, the lost in field energy per unit time can be written as[38]

$$R = \sum_{n,m}\rho_n w_{mn}\omega_{mn}, \tag{55}$$

where $\rho_n=\exp[(F-E_n)/T]$ is the Gibbs distribution function, E_n—energy levels of the system, F—its free energy, $\omega_{mn}=E_m-E_n$, and w_{mn} is the probability of the transition $n\to m$ (per unit time) due to the interaction with the field. By dropping the diamagnetic current and assuming the field to be monochromatic, the interaction can be written as

$$\frac{1}{2}\left(\frac{e}{c}\mathbf{v}\cdot\mathbf{A}_\omega+\frac{g\mu_B}{2}\boldsymbol{\sigma}\cdot\mathbf{B}_\omega\right)e^{-i\omega t}+\text{c.c.} \tag{56}$$

Then

$$w_{mn}=\frac{2\pi}{4}\left\{\left|\frac{e}{i\omega}\mathbf{v}_{mn}\cdot\mathbf{E}_\omega+\frac{g\mu_B}{2}\boldsymbol{\sigma}_{mn}\cdot\mathbf{B}_\omega\right|^2\delta(\omega-\omega_{mn})+\left|\frac{ie}{\omega}\mathbf{v}_{mn}\cdot\mathbf{E}^*_\omega+\frac{g\mu_B}{2}\boldsymbol{\sigma}_{mn}\cdot\mathbf{B}^*_\omega\right|^2\delta(\omega+\omega_{mn})\right\}, \tag{57}$$

where $\mathbf{E}_\omega=(i\omega/c)\mathbf{A}_\omega$ and matrix elements of spin and velocity operators are taken between the exact eigenstates of the unperturbed system. The part of w_{mn} bilinear in the electric and magnetic fields after some manipulation can be put in the form

$$ie\left(\frac{g\mu_B}{8}\right)\int_{-\infty}^{\infty}dte^{i\omega t}\sum_{n,m}\rho_n\{-(n|\boldsymbol{\sigma}(t)\cdot\mathbf{B}^*_\omega|m)(m|\mathbf{v}\cdot\mathbf{E}_\omega|n)+(n|\mathbf{v}\cdot\mathbf{E}_\omega|m)(m|\boldsymbol{\sigma}(t)\cdot\mathbf{B}^*_\omega|n)+(n|\mathbf{v}(t)\cdot\mathbf{E}^*_\omega|m)\times(m|\boldsymbol{\sigma}\cdot\mathbf{B}_\omega|n)-(n|\boldsymbol{\sigma}\cdot\mathbf{B}_\omega|m)(m|\mathbf{v}(t)\cdot\mathbf{E}^*_\omega|n)\}, \tag{58}$$

what is nothing but

$$ie\left(\frac{g\mu_B}{8}\right)\int_{-\infty}^{\infty}dte^{i\omega t}\{\langle[\mathbf{v}(t)\cdot\mathbf{E}^*_\omega,\boldsymbol{\sigma}\cdot\mathbf{B}_\omega]\rangle-\langle[\boldsymbol{\sigma}(t)\cdot\mathbf{B}^*_\omega,\mathbf{v}\cdot\mathbf{E}_\omega]\rangle\}, \tag{59}$$

where the square and angle brackets denote the commutator and the averaging over the canonical ensemble. By employing the real-time formalism[30] (instead of the thermal ones used in Sec. II B), one can obtain the representation of $\hat{\theta}(\omega)$ and $\hat{\gamma}(\omega)$ through the retarded correlation functions

$$\theta_{ij}(\omega)=\frac{e}{\omega}\left(\frac{g\mu_B}{2}\right)\int_0^\infty dte^{i(\omega+i0)t}\langle[v_i(t),\sigma_j]\rangle, \tag{60}$$

$$\gamma_{ij}(\omega)=\frac{e}{\omega}\left(\frac{g\mu_B}{2}\right)\int_0^\infty dte^{i(\omega+i0)t}\langle[\sigma_i(t),v_j]\rangle. \tag{61}$$

By making use of Eqs. (60) and (61), it is not difficult to see that Eq. (59) is identical with R_{EB} which follows from Eq. (54). Thus one may treat the magnetoelectric contribution to the absorption coefficient as a result of an interference between quantum transitions induced by the electric and magnetic fields.

In the absence of an external magnetic field, when Eqs. (50) and (51) are applicable, the expression for the magnetoelectric term takes the form

$$\omega\gamma'(\omega)\text{Im}(\mathbf{E}^*_\omega\times\mathbf{B}_\omega\cdot\mathbf{c}). \tag{62}$$

In the case of a free electromagnetic wave incident on a 2D electron system $\text{Im}(\mathbf{E}^*_\omega\times\mathbf{B}_\omega\cdot\mathbf{c})=0$. But if the microwave radiation is absorbing by a bulk conductor, within which the wave vector of the radiation has an imaginary part, the magnetoelectric contribution to the power loss should be finite so that the absorption acquires a dependence on the angle between the Poynting vector of the radiation and the polar axis. The magnetoelectric contribution is also finite for a 2D electron system subject to an external magnetic field, as it will be shown in Sec. VI.

III. SPIN SUSCEPTIBILITY

A. Bethe-Salpeter kernel

In this section, we evaluate the spin susceptibility of the system defined by Hamiltonian (2) following the lines of Sec. II C. Only the logic of the evaluation is given; all calculations are placed in Appendix B. As usually, we use the coordinate representation dealing with the Landau quantization. In this representation, the susceptibility of the system in the presence of disorder, as a function of frequency and wave vector, is given by[30]

$$\chi^{(dyn)}_{ij}(\mathbf{q},\omega)=-\left(\frac{g}{2}\mu_B\right)^2 i\omega\int_{-\infty}^{\infty}\frac{d\epsilon}{2\pi}N'(\epsilon,\omega)\times\sigma^i_{\delta\alpha}\Pi_{\alpha\beta,\gamma\delta}(\mathbf{q},\omega;\epsilon)\sigma^j_{\beta\gamma}, \tag{63}$$

where $N'(\epsilon,\omega)=\frac{1}{\omega}[f_F(\epsilon)-f_F(\epsilon+\omega)]$, $f_F(\epsilon)=[\exp(\frac{\epsilon-\mu}{T})+1]^{-1}$ is the Fermi distribution function while

$$\Pi_{\alpha\beta,\gamma\delta}(\mathbf{q},\omega;\epsilon)=\int_{\mathbf{r}-\mathbf{r}'}e^{-i\mathbf{q}\cdot(\mathbf{r}-\mathbf{r}')}\Pi_{\alpha\beta,\gamma\delta}(\mathbf{r},\mathbf{r}';\epsilon,\omega) \tag{64}$$

is the Fourier transform of the joint propagator of a particle with energy $\epsilon+\omega$ and a hole with energy ϵ in the presence of disorder

$$\Pi_{\alpha\beta,\gamma\delta}(\mathbf{r},\mathbf{r}';\epsilon,\omega)=\langle G^R_{\alpha\beta}(\mathbf{r},\mathbf{r}';\epsilon+\omega)G^A_{\gamma\delta}(\mathbf{r}',\mathbf{r};\epsilon)\rangle, \tag{65}$$

where G^R and G^A are the advanced and retarded single-particle Green's functions corresponding to Hamiltonian (2), $\langle\,\rangle$ implies impurity averaging, and the fact[39] is used that $\hat{\Pi}(\mathbf{r},\mathbf{r}';\epsilon,\omega)$ depends only on the relative coordinate $\mathbf{r}-\mathbf{r}'$. Everywhere below, the wave vector $\mathbf{q}$ is assumed to be zero. The particle-hole propagator $\hat{\Pi}$ is the sum of an infinitive series

$$\Pi_{\alpha\beta,\gamma\delta}(\mathbf{r},\mathbf{r}';\epsilon,\omega)=\Pi^{(0)}_{\alpha\beta,\gamma\delta}(\mathbf{r},\mathbf{r}';\epsilon,\omega)+n_{imp}U^2\int_{\mathbf{r}_1}\Pi^{(0)}_{\alpha\kappa,\rho\delta}(\mathbf{r},\mathbf{r}_1;\epsilon,\omega)\times\Pi^{(0)}_{\kappa\beta,\gamma\rho}(\mathbf{r}_1,\mathbf{r}';\epsilon,\omega)+\cdots, \tag{66}$$

where

$$\Pi^{(0)}_{\alpha\beta,\gamma\delta}(\mathbf{r},\mathbf{r}';\epsilon,\omega)=\langle G^R_{\alpha\beta}(\mathbf{r},\mathbf{r}';\epsilon+\omega)\rangle\langle G^A_{\gamma\delta}(\mathbf{r}',\mathbf{r};\epsilon)\rangle. \tag{67}$$

In the Feynman-diagram language, the series [Eq. (66)] represents the sum of the so-called ladder diagrams. This sum satisfies the Bethe-Salpeter equation

$$\Pi_{\alpha\beta,\gamma\delta}(\mathbf{r},\mathbf{r}';\epsilon,\omega) = \Pi^{(0)}_{\alpha\beta,\gamma\delta}(\mathbf{r},\mathbf{r}';\epsilon,\omega) + n_{imp}U^2\int_{\mathbf{r}_1}\Pi^{(0)}_{\alpha\kappa,\rho\delta}(\mathbf{r},\mathbf{r}_1;\epsilon,\omega) \times\Pi_{\kappa\beta,\gamma\rho}(\mathbf{r}_1,\mathbf{r}';\epsilon,\omega). \quad (68)$$

The irreducible part of $\hat{\Pi}$, $\hat{\Pi}^{(0)}(\mathbf{r},\mathbf{r}';\epsilon,\omega)$, which is the kernel of the integral Eq. (68), can be considered as the basic building block of the theory.[40] It is the point where obstacles to incorporating diamagnetism into the theory appear.

First of all note that the effect of impurity scattering can be considered in the framework of a technique developed in the work,[41] where the scattering from a single impurity site is treated in lowest Born approximation while effects of level broadening are accounted for self-consistently. As a result, the self-energy of the impurity averaged Green's function acquires an energy dependent imaginary part, $\pm i/2\tau(\epsilon)$, independent of the energy of Landau level. As the presumably weak SO coupling plays a negligible role in the one-electron decay at $\alpha p_F\tau \ll 1$, the decay may be calculated regardless of H_{so}. Then $\tau(\epsilon)$ and the density of states for a single spin, $N(\epsilon)=-\frac{p}{\pi}\Sigma_n \mathrm{Im}[\epsilon-(n+\frac{1}{2})\omega_c-\frac{i}{2\tau(\epsilon)}]^{-1}$, where $p=\frac{1}{2\pi\lambda^2}$ is the Landau-level degeneracy and $\lambda=\frac{c}{eB_{(0)}}$ is the magnetic length, are related via the equation[22]

$$\frac{1}{\tau(\epsilon)} = 2\pi N(\epsilon)n_{imp}U^2. \quad (69)$$

By virtue of this equation, $N(\epsilon)$ takes on a nonzero value within the intervals $(-\frac{1}{2\tau},\frac{1}{2\tau})$ about each Landau level. The detailed form of τ can be obtained from Ref. 41; when $\omega_c\tau_0 \gg 1$, $\tau \cong (\pi\tau_0/2\omega_c)^{1/2}$. Further, taking into account that the magnetic field is supposed to be adjusted to make $|(n+\frac{1}{2}\omega_c)-\epsilon_F|\tau_0 \ll 1$ for some integer $n=n_0$, the ϵ integration in Eq. (63) is essentially limited by the factor $N'(\epsilon,\omega)$ and the condition $\omega_s\tau_0 \ll 1$, all quantities involved are evaluated at the Fermi energy. Therefore, we can set $\tau=\tau(\epsilon_F)$, drop the dependence of $\hat{\Pi}^{(0)}$ on ϵ, and put $T=0$ in Eq. (63) assuming $T\tau \ll 1$.

Now, according to directions given in Sec. II C, we expand the kernel $\hat{\Pi}^{(0)}(\mathbf{r},\mathbf{r}';\epsilon,\omega)$ in series in the small parameters $\alpha p_F\tau$ and $\omega_s\tau$ or, what is the same, in the spin-orbit coupling H_{so} and the Zeeman interaction H_Z. As in the case of zero magnetic field [compare with Eq. (21)], it is convenient to deal with the dimensionless kernel

$$(\delta\kappa|T(\omega)|\alpha\beta) \overset{def}{=} T_{\delta\beta,\alpha\kappa}(\omega) = n_{imp}U^2\Pi^{(0)}_{\delta\beta,\alpha\kappa}(0,\omega;\epsilon_F). \quad (70)$$

The unperturbed kernel $\hat{T}^{(0)}$ has the form

$$(\alpha\delta|T^{(0)}(\omega)|\gamma\beta) = n_{imp}U^2\int_{\mathbf{r}_1-\mathbf{r}_2} G^{R(0)}_{\alpha\beta}(\mathbf{r}_1,\mathbf{r}_2;\epsilon_F+\omega) \times G^{A(0)}_{\gamma\delta}(\mathbf{r}_2,\mathbf{r}_1;\epsilon_F). \quad (71)$$

The spectral decomposition of the impurity averaged Green's function corresponding to the Hamiltonian H_0 is

$$G^{R(A)}_{(0)\alpha\beta}(\mathbf{r}_1,\mathbf{r}_2;\epsilon) = \delta_{\alpha\beta}\sum_{n,l}\langle\mathbf{r}_1|n,l\rangle\langle n,l|\mathbf{r}_2\rangle G^{R(A)}_n(\epsilon),$$

$$G^{R(A)}_n(\epsilon) = \frac{1}{\epsilon-\epsilon_n \pm i/2\tau}. \quad (72)$$

Here $\langle\mathbf{r}|n,l\rangle$ is the eigenvector of H_0 corresponding to the eigenvalue $\epsilon_n=\omega_c(n+1/2)$. Its explicit form is given in Appendix B. Substituting Eq. (72) into Eq. (71), we obtain

$$(\alpha\delta|T^{(0)}(\omega)|\gamma\beta) \cong \delta_{\alpha\beta}\delta_{\gamma\delta}(1+i\omega\tau), \quad (73)$$

which has the same form as in the zero-magnetic-field case [see Eq. (33)] and depends on the cyclotron frequency only through τ.

Now consider contributions to the kernel $\hat{T}$ which are of the first order in small parameters $\omega_s\tau$ or $\alpha p_F\tau$. The correction to $G^{R(A)}_{(0)}(\mathbf{r}_1,\mathbf{r}_2;\epsilon)_{\alpha\beta}$ due to H_Z has the form

$$G^{R(A)}_{(1.Z)\alpha\beta}(\mathbf{r}_1,\mathbf{r}_2;\epsilon) = \int_{\mathbf{r}'}[\hat{G}^{R(A)}_{(0)}(\mathbf{r}_1,\mathbf{r}';\epsilon)\hat{H}_Z\hat{G}^{R(A)}_{(0)}(\mathbf{r}',\mathbf{r}_2;\epsilon)]_{\alpha\beta}. \quad (74)$$

The corresponding correction to $\hat{T}$ is

$$(\alpha\delta|T^{(1)}_{par}|\gamma\beta) = n_{imp}U^2\int_{\mathbf{r}_1-\mathbf{r}_2}[G^{R(1.Z)}_{\alpha\beta}(\mathbf{r}_1,\mathbf{r}_2;\epsilon_F+\omega)G^{A(0)}_{\gamma\delta}(\mathbf{r}_2,\mathbf{r}_1;\epsilon_F) + G^{R(0)}_{\alpha\beta}(\mathbf{r}_1,\mathbf{r}_2;\epsilon_F+\omega)G^{A(1.Z)}_{\gamma\delta}(\mathbf{r}_2,\mathbf{r}_1;\epsilon_F)]. \quad (75)$$

Here, because the correction is already proportional to the small parameter $\omega_s\tau$, one can neglect the small parameter $\omega\tau$ and set $\omega=0$. A calculation yields [see Appendix B]

$$(\alpha\delta|T^{(1)}_{par}|\gamma\beta) = \frac{i}{2}(\omega_s\tau)[(\mathbf{h}\cdot\boldsymbol{\sigma})_{\alpha\beta}\delta_{\gamma\delta}-\delta_{\alpha\beta}(\mathbf{h}\cdot\boldsymbol{\sigma})_{\gamma\delta}], \quad (76)$$

which also depends on ω_c only through τ.

The first correction to $G^{R(A)}_{(0)}(\mathbf{r}_1,\mathbf{r}_2;\epsilon)$ due to H_{so} is

$$G^{R(A)}_{(1,so)}(\mathbf{r}_1,\mathbf{r}_2;\epsilon) = \int_{\mathbf{r}'}G^{R(A)}_{(0)}(\mathbf{r}_1,\mathbf{r}';\epsilon)H_{so}(\mathbf{r}')G^{R(A)}_{(0)}(\mathbf{r}',\mathbf{r}_2;\epsilon). \quad (77)$$

The corresponding correction to $\hat{T}$

$$(\alpha\delta|T^{(1)}_{so}|\gamma\beta) = n_{imp}U^2\int_{\mathbf{r}_1-\mathbf{r}_2}[G^{R(1,so)}_{\alpha\beta}(\mathbf{r}_1,\mathbf{r}_2,\epsilon_F)G^{A(0)}_{\gamma\delta}(\mathbf{r}_2,\mathbf{r}_1,\epsilon_F) + G^{R(0)}_{\alpha\beta}(\mathbf{r}_1,\mathbf{r}_2,\epsilon_F)G^{A(1,so)}_{\gamma\delta}(\mathbf{r}_2,\mathbf{r}_1,\epsilon_F)] \quad (78)$$

equals zero just as in the zero-magnetic-field case.

Now consider corrections of the second order. The correction to $G^{R(A)}_{(0)}(\mathbf{r}_1,\mathbf{r}_2;\epsilon)$ due to H_{so} has the form

$$G^{R(A)}_{(2)so}(\mathbf{r}_1,\mathbf{r}_2) = \int_{\mathbf{r}}\int_{\mathbf{r}'} G^{R(A)}_{(0)}(\mathbf{r}_1,\mathbf{r})H_{so}(\mathbf{r})G^{R(A)}_{(0)}(\mathbf{r},\mathbf{r}') \times H_{so}(\mathbf{r}')G^{R(A)}_{(0)}(\mathbf{r}',\mathbf{r}_2). \quad (79)$$

Accordingly, the second-order correction to $\hat{T}^{(0)}$ due to H_{so} is

$$\hat{T}^{(2)}_{so} = \hat{P}_{(1.1)} + \hat{P}_{(2.0)} + \hat{P}_{(0.2)},$$

$$(\alpha\delta|P_{(1.1)}|\gamma\beta) = n_{imp}U^2\int_{\mathbf{r}_1-\mathbf{r}_2} G^{R(1,so)}_{\alpha\beta}(\mathbf{r}_1,\mathbf{r}_2)G^{A(1,so)}_{\gamma\delta}(\mathbf{r}_2,\mathbf{r}_1),$$

$$(\alpha\delta|P_{(2.0)}|\gamma\beta) = n_{imp}U^2\int_{\mathbf{r}_1-\mathbf{r}_2} G^{R(2,so)}_{\alpha\beta}(\mathbf{r}_1,\mathbf{r}_2)G^{A(0)}_{\gamma\delta}(\mathbf{r}_2,\mathbf{r}_1),$$

$$(\alpha\delta|P_{(0.2)}|\gamma\beta) = n_{imp}U^2\int_{\mathbf{r}_1-\mathbf{r}_2} G^{R(0)}_{\alpha\beta}(\mathbf{r}_1,\mathbf{r}_2)G^{A(2,so)}_{\gamma\delta}(\mathbf{r}_2,\mathbf{r}_1). \quad (80)$$

A calculation yields [see Appendix B]

$$(\alpha\delta|P_{(1.1)}|\gamma\beta) = \frac{2(\alpha p_F\tau)^2}{1+(\omega_c\tau)^2}(s^+_{\alpha\beta}s^-_{\gamma\delta} + s^-_{\alpha\beta}s^+_{\gamma\delta}),$$

$$(\alpha\delta|P_{(2.0)}|\gamma\beta) = -2(\alpha p_F\tau)^2[(1+i\omega_c\tau)^{-1}\Pi^{(u)}_{\alpha\beta} + (1-i\omega_c\tau)^{-1}\Pi^{(d)}_{\alpha\beta}]\delta_{\gamma\delta},$$

$$(\alpha\delta|P_{(2.0)}|\gamma\beta) = -2(\alpha p_F\tau)^2\delta_{\alpha\beta}[(1-i\omega_c\tau)^{-1}\Pi^{(u)}_{\gamma\delta} + (1+i\omega_c\tau)^{-1}\Pi^{(d)}_{\gamma\delta}], \quad (81)$$

where $s_\pm = (\sigma_x \pm \sigma_y)/2$, $\Pi^{(u,d)} = (1 \pm \mathbf{h}_\perp \cdot \boldsymbol{\sigma})/2$, and $\mathbf{h}_\perp = \mathbf{B}^{(0)}_\perp/|\mathbf{B}^{(0)}_\perp|$. Thus, by making use of the equality

$$s^+_{\gamma\delta}s^-_{\alpha\beta} + s^-_{\gamma\delta}s^+_{\alpha\beta} = \frac{1}{2}(\mathbf{c}\times\boldsymbol{\sigma})_{\gamma\delta}\cdot(\mathbf{c}\times\boldsymbol{\sigma})_{\alpha\beta}, \quad (82)$$

we get

$$(\alpha\delta|T^{(2)}_{so}|\gamma\beta) = -\frac{(\alpha p_F\tau)^2}{1+(\omega_c\tau)^2}\{(\mathbf{c}\times\boldsymbol{\sigma})_{\gamma\delta}\cdot(\mathbf{c}\times\boldsymbol{\sigma})_{\alpha\beta} + 2\delta_{\gamma\delta}\delta_{\alpha\beta} + i(\omega_c\tau)[(\mathbf{h}_\perp\cdot\boldsymbol{\sigma})_{\gamma\delta}\delta_{\alpha\beta} - \delta_{\gamma\delta}(\mathbf{h}_\perp\cdot\boldsymbol{\sigma})_{\alpha\beta}]\}. \quad (83)$$

Note that the last term in Eq. (83) has the same spin structure as $\hat{T}^{(1)}_{par}$ from Eq. (76). This fact shows that owing to the BSOC the cyclotron motion can contribute to the same physical processes which in systems without the BSOC are controlled only by the Zeeman interaction. A contribution to $\hat{T}$ of order $\eta(\omega_s\tau)$, i.e., bilinear in H_{so} and H_Z, equals zero [see Appendix B]. Contributions quadratic in ω_s are finite but can be ignored since they are only a corrections on the order of $(\omega_s\tau)^2$ to $\hat{T}^{(0)}$. Contributions of order $\eta^2(\omega_s\tau)$, i.e., linear in H_Z and quadratic in H_{so}, are also finite but small at $\eta \ll 1$.

B. Spin vertex

Instead of the particle-hole propagator $\hat{\Pi}(\omega)$, which is an object of four spin indexes, it is simpler to deal with the renormalized spin vertex $\boldsymbol{\Sigma}$, which is an object of two spin indexes. The equation for the left spin vertex has the same form as in the zero-magnetic-field case [see Eq. (47)]

$$\boldsymbol{\Sigma}^{(l)}_{\gamma\beta}(\omega) = \boldsymbol{\sigma}_{\gamma\beta} + \boldsymbol{\Sigma}^{(l)}_{\delta\alpha}(\omega)(\alpha\delta|T(\omega)|\gamma\beta). \quad (84)$$

In the Feynman-diagram language, it turns up while one considers ladder diagrams for $\hat{\chi}$ as a renormalization of the left σ vertex. The right renormalized spin vertex, which turns up while one considers ladder diagrams for $\hat{\chi}$ as a renormalization of the right σ vertex, also can be introduced. It obeys the equation

$$\boldsymbol{\Sigma}^{(r)}_{\alpha\delta}(\omega) = \boldsymbol{\sigma}_{\alpha\delta} + (\alpha\delta|T(\omega)|\gamma\rho)\boldsymbol{\Sigma}^{(r)}_{\rho\gamma}(\omega), \quad (85)$$

which is an analog of Eq. (22). The susceptibility is expressed through the solution of Eq. (84) as

$$\chi_{ij}(\omega)^{(dyn)} = \frac{i\omega}{2\pi}\left(\frac{g\mu_B}{2}\right)^2(n_{imp}|U|^2)^{-1}\Sigma^{(l)i}_{\delta\alpha}(\omega)(\alpha\delta|T(\omega)|\gamma\rho)\sigma^j_{\rho\gamma} = i\omega\tau N(\epsilon_F)\left(\frac{g\mu_B}{2}\right)^2\mathrm{Tr}\{\Sigma^{(l)}_i(\omega)\hat{T}(\omega)\sigma_j\}. \quad (86)$$

To simplify the treatment of Eq. (84), which plays the role of the quantum kinetic equation and hence is the central equation of the theory, we perform two transformation. (i) The first transformation, just as in the zero-magnetic-field case, consists in a rearrangement of spin indexes of the kernel $\hat{T}$ so that, for example, a pair of indices, γ, ρ, by means of which the kernel is connected with σ_j in Eq. (86), appears at one spin matrix and the same is true with respect of another pair of indices α, δ, by means of which the kernel is connected with $\boldsymbol{\Sigma}^{(l)}(\omega)$. As a result, we get

$$T(\omega) = T_{relax} + T_{prec}, \quad (87)$$

where

$$(\alpha\delta|T_{relax}|\gamma\beta) \cong \frac{a}{2}\delta_{\alpha\delta}\delta_{\gamma\beta} + \frac{b}{2}(\mathbf{c}\cdot\boldsymbol{\sigma})_{\alpha\delta}(\mathbf{c}\cdot\boldsymbol{\sigma})_{\gamma\beta} + \frac{u}{2}(\mathbf{c}\times\boldsymbol{\sigma})_{\alpha\delta}(\mathbf{c}\times\boldsymbol{\sigma})_{\gamma\beta} \quad (88)$$

with

$$a = 1 + i\zeta, \quad b = 1 + i\zeta - \bar{\eta}^2,$$

$$u = 1 + i\zeta - \frac{\bar{\eta}^2}{2},$$

$$\zeta = \omega\tau, \quad \bar{\eta} = (2\alpha p_F\tau)[1+(\omega_c\tau)^2]^{-1/2} \quad (89)$$

while

$$(\alpha\delta|T_{prec}|\gamma\beta) = \frac{1}{2}\left[(\omega_s\tau)h^i - \frac{\overline{\eta}^2}{2}(\omega_c\tau)h^i_\perp\right]e_{ijk}\sigma^j_{\alpha\delta}\sigma^k_{\gamma\beta}. \quad (90)$$

It will be seen that $\hat{T}_{relax}$ is responsible for the spin relaxation, whereas $\hat{T}_{prec}$ for the spin precession. It is convenient to join together both terms in $\hat{T}_{prec}$ to obtain

$$(\alpha\delta|T_{prec}|\gamma\beta) = -\frac{\Omega}{2}e_{ijk}t^i\sigma^j_{\alpha\delta}\sigma^k_{\gamma\beta}, \quad (91)$$

where[42]

$$\Omega = \omega_{res}\tau, \omega_{res} = \left\{\omega_s^2 - \overline{\eta}^2\omega_s\omega_c|(\mathbf{c}\cdot\mathbf{h})| + \left[\frac{\overline{\eta}^2}{2}\omega_c\right]^2\right\}^{1/2},$$

$$\mathbf{t} \cong \mathbf{h} + \left(\overline{\eta}^2\frac{\omega_c}{2\omega_s}\right)[\mathbf{h}|\mathbf{c}\cdot\mathbf{h}| - \mathbf{c}\ \mathrm{sgn}(\mathbf{c}\cdot\mathbf{h})]. \quad (92)$$

(ii) The second transformation is a transition to a new set of spin matrices. Let the orthogonal 2D vectors $(\hat{x},\hat{y})$ lie in the plane of the structure and the vector $\hat{z}$ is perpendicular to the plane. Instead of the standard set of Pauli matrices $(\sigma_x,\sigma_y,\sigma_z)$, which can be considered as projections of the spin-vector $\boldsymbol{\sigma}$ on the basis $\hat{x}$, $\hat{y}$, $\hat{z}$, we introduce a new set which is "bound" to the external magnetic field $\mathbf{B}_{(0)}$ and the polar axis $\mathbf{c}$. At a general direction of $\mathbf{B}_{(0)}$ with respect of the plane of the structure, we define

$$(\tau_1,\tau_2,\tau_3) \stackrel{def}{=} (\boldsymbol{\sigma}\cdot\mathbf{f}_{(1)},\boldsymbol{\sigma}\cdot\mathbf{f}_{(2)},\boldsymbol{\sigma}\cdot\mathbf{f}_{(3)}),$$

$$(\mathbf{f}_{(1)},\mathbf{f}_{(2)},\mathbf{f}_{(3)}) = \left[\mathbf{t}, \frac{\mathbf{c}\times\mathbf{t}}{\sqrt{1-(\mathbf{c}\cdot\mathbf{t})^2}}, \frac{\mathbf{c}-\mathbf{t}(\mathbf{c}\cdot\mathbf{t})}{\sqrt{1-(\mathbf{c}\cdot\mathbf{t})^2}}\right]. \quad (93)$$

Because unit vectors $\mathbf{f}_{(1)}$, $\mathbf{f}_{(2)}$, $\mathbf{f}_{(3)}$ form the orthogonal right-handed basis, just as the standard one, the commutative relations for matrices τ_1, τ_2, τ_3 are the same as those for matrices σ_x, σ_y, σ_z. In terms of the new set, the kernel $\hat{T}(\omega)$ has the form

$$(\alpha\delta|T(\omega)|\gamma\beta) = \frac{a}{2}\delta_{\alpha\delta}\delta_{\gamma\beta} + \frac{1}{2}[(bc^2+us^2)\tau^1_{\alpha\delta}\tau^1_{\gamma\beta} + u\tau^2_{\alpha\delta}\tau^2_{\gamma\beta} + (bs^2+uc^2)\tau^3_{\alpha\delta}\tau^3_{\gamma\beta} + (b-u)cs(\tau^1_{\alpha\delta}\tau^3_{\gamma\beta} + \tau^3_{\alpha\delta}\tau^1_{\gamma\beta})] - \frac{\Omega}{2}(\tau^2_{\alpha\delta}\tau^3_{\gamma\beta} - \tau^3_{\alpha\delta}\tau^2_{\gamma\beta}), \quad (94)$$

where $c=(\mathbf{c}\cdot\mathbf{h})$ and $s=\sqrt{1-(\mathbf{c}\cdot\mathbf{h})^2}$.

Since the kernel $\hat{T}(\omega)$ does not entangle the scalar and spin channels, i.e., does not contain terms of the form $\delta_{\alpha\delta}\tau_{\gamma\beta}$ and $\delta_{\gamma\beta}\tau_{\alpha\delta}$, the solution of Eq. (84) should be a linear combination of τ matrices. This circumstance as well as the vector character of $\boldsymbol{\Sigma}$ suggests the following ansatz

$$\boldsymbol{\Sigma}^{(l)}(\omega) = \sum_{s,m=1}^{3}\mathbf{f}_{(m)}V_{ms}\tau_s \quad (95)$$

with the help of which Eq. (84) reduces to the equation for the matrix V_{ms}

$$\sum_{s=1}^{3}V_{is}R_{sn} = \delta_{in}, \quad (96)$$

where R_{sn} is the 3×3 matrix

$$R = \begin{pmatrix} A & 0 & P \\ 0 & B & \Omega \\ P & -\Omega & C \end{pmatrix} \quad (97)$$

with elements

$$A = -i\zeta + \frac{\overline{\eta}^2}{2}(1+c^2),\ B = -i\zeta + \frac{\overline{\eta}^2}{2},$$

$$C = -i\zeta + \frac{\overline{\eta}^2}{2}(2-c^2),\ P = cs\frac{\overline{\eta}^2}{2}. \quad (98)$$

Thus

$$V_{is} = (R)^{-1}_{is} = \frac{1}{\det R}\begin{pmatrix} BC+\Omega^2 & -P\Omega & -BP \\ P\Omega & AC-P^2 & -A\Omega \\ -BP & A\Omega & AB \end{pmatrix}, \quad (99)$$

where $\det R = ABC + A\Omega^2 - BP^2$.

In the kernel $\hat{T}(\omega)$ entering Eq. (86), one may neglect terms proportional to the small parameters $\omega\tau$, $\omega_s\tau$, and $\overline{\eta}$, i.e., one may substitute $T_{\alpha\beta,\gamma\delta}(\omega) \cong \delta_{\alpha\beta}\delta_{\gamma\delta}$. Then Eq. (86) takes the form

$$\chi^{(dyn)}_{ij}(\omega) \simeq i\omega\tau\frac{\pi}{m}N(\epsilon_F)\chi_0\ \mathrm{Tr}\{\Sigma^{(l)}_i(\omega)\sigma_j\}. \quad (100)$$

where $\chi_0 = \frac{m}{\pi}(\frac{g\mu_B}{2})^2$ is the static susceptibility of 2D degenerate electron gas and

$$\mathrm{Tr}\{\Sigma^{(l)}_i\sigma_j\} = 2\sum_{nm}R^{-1}_{nm}f^i_{(n)}f^j_{(m)}. \quad (101)$$

Equations (100) and (101) settle the problem of calculation of the spin susceptibility under conditions $\omega_s\tau \ll 1$ and $\overline{\eta} \ll 1$. In the following we assume that the more restrictive sharp-resonance condition [Eq. (4)] or, more precisely, $\overline{\eta}^2/\Omega \ll 1$ is satisfied. The determinant of the matrix R is a polynomial of the third order of argument ζ and hence it has three roots. A simple analysis shows that up to corrections on the order of $(\overline{\eta}^2/\Omega)^2$ the roots are

$$\zeta_0 \cong -i\frac{\overline{\eta}^2}{2}(1+c^2),$$

$$\zeta_\pm \cong \pm\omega_{res} - i\frac{\overline{\eta}^2}{2}(3-c^2), \quad (102)$$

where

$$\omega_{res} = \frac{\Omega}{\tau} = \left\{\omega_s^2 - \overline{\eta}^2\omega_s\omega_c|(\mathbf{c}\cdot\mathbf{h})| + \left[\frac{\overline{\eta}^2}{2}\omega_c\right]^2\right\}^{1/2}. \quad (103)$$

In a vicinity of the pole ζ_0,

$$R^{-1} \cong \frac{i}{\zeta - \zeta_0}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{104}$$

Accordingly, at low frequencies, the spin susceptibility has the form

$$\chi_{ij}(\omega) \cong \chi_0 \frac{2\pi}{m} N(\epsilon_F) \frac{-\omega}{\omega + \dfrac{i}{T_1}} t_i t_j,$$

$$\frac{1}{T_1} = \frac{\bar{\eta}^2}{2\tau}(1 + c^2). \tag{105}$$

In a vicinity of the pole ζ_+

$$R^{-1} \cong \frac{i/2}{\zeta - \zeta_+}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -i \\ 0 & i & 1 \end{pmatrix}. \tag{106}$$

Hence

$$\mathrm{Tr}\{\Sigma_i^{(l)}\sigma_j\} \cong \frac{i}{\zeta - \zeta_+}[(f^i_{(2)}f^j_{(2)} + f^i_{(2)}f^j_{(2)}) - i(f^i_{(2)}f^j_{(3)} - f^i_{(3)}f^j_{(2)})]. \tag{107}$$

By making use of the equalities

$$f_{(2)i}f_{(2)j} + f_{(2)i}f_{(2)j} = \delta_{ij} - f_{(1)i}f_{(1)j},$$

$$f_{(2)i}f_{(3)j} - f_{(3)i}f_{(2)j} = e_{ijk}f_{(1)k}, \tag{108}$$

we get near the frequency of the ESR

$$\chi_{ij}(\omega) \cong \chi_0 \frac{\pi}{m} N(\epsilon_F) \frac{-\omega}{\omega - \omega_{res} + \dfrac{i}{T_2}} (\delta_{ij} - t_i t_j - i e_{ijk} t_k),$$

$$\frac{1}{T_2} = \frac{\bar{\eta}^2}{4\tau}(3 - c^2). \tag{109}$$

The matrix structure of the numerator of this expression shows that it is the **t** direction defined by Eq. (92) rather than **h** direction is the axis of the spin precession—the magnetic term in the energy absorption $R_B = \frac{\omega}{2}(\mathbf{B}^*_\omega \cdot \hat{\chi}''(\omega) \cdot \mathbf{B}_\omega)$ vanishes at $\mathbf{B}_\omega \| \mathbf{t}$. Thus the cyclotron motion changes the direction of the spin precession axis when the external magnetic field $B_{(0)}$ deviates from the **c** direction. In a particular case of $\mathbf{B}^{(0)} \| \mathbf{c}$, the longitudinal relaxation time T_1 of Eq. (105) and the transverse relaxation time T_2 of Eq. (109) reduce to the corresponding expressions derived in Ref. 25. At $\omega_c \tau_0 \gg 1$, when the discreteness of the electron energy spectrum is pronounced as much as possible and hence the description of the conduction-electron-spin resonance (CESR) should resemble one in terms of transitions between quantum states, Eq. (103) at $\mathbf{B}^{(0)} \| \mathbf{c}$ gives $\omega_{res} \cong \omega_s - 4m\alpha^2 \frac{\epsilon_F}{\omega_c}$. As $\omega_s \ll \omega_c$, this expression is in accord with the spin splitting $\omega_s - 4n_0 m\alpha^2 \frac{\omega_c}{\omega_c - \omega_s}$ of the Landau level with $n_0 \simeq \frac{\epsilon_F}{\omega_c} \gg 1$, which follows from an explicit solution of $H_0 + H_{so} + H_Z$.[15] All above, we considered the system of carriers with negative charge and positive sign of g in Eq. (2), i.e., as in the case of free electrons. In the case of $g < 0$, which can be considered quite analogously, we come to the same Eq. (109) but with the resonant frequency $\omega_{res} = \{\omega_s^2 + \bar{\eta}^2 \omega_s \omega_c |(\mathbf{c} \cdot \mathbf{h})| + [\frac{\bar{\eta}^2}{2}\omega_c]^2\}^{1/2}$, where $\omega_s = |g|\mu_B B_{(0)}$. Thus, depending on the sign of g factor with respect to the sign of charge carriers, the cyclotron motion can impede or maintain the spin precession. Note that both the relaxation time and the shift of the resonance frequency induced by the cyclotron motion are controlled by the same parameter $\bar{\eta}$. The shift was missed in Ref. 25. The results presented are valid at $\omega_s \tau \ll 1$, $\bar{\eta} \ll 1$, and $\bar{\eta}^2/\Omega \ll 1$. The second criterion reduces to the known criterion of applicability of the work[19] $\eta_0 \ll 1$ when $\omega_c \tau_0 \ll 1$ and to $\frac{\alpha p_F}{\omega_c} \ll 1$ when $\omega_c \tau_0 \ll 1$. Thus results of Ref. 26 do not require the fulfillment of the inequality $\frac{\alpha p_F}{\omega_c} \ll 1$ at small external fields. It is interesting to note that in the case of a structure with zero g factor, the resonant frequency does not vanishes. According to Eq. (103), it becomes $\omega_{res} = \frac{\bar{\eta}^2}{2}\omega_c$. Thus the cyclotron motion is capable of maintaining the spin precession alone, without the Zeeman interaction. The parameter determining the sharpness of the resonance $\omega_{res} T_2 \simeq \omega_c \tau$ can exceed unity.

IV. CROSS SUSCEPTIBILITIES

In this section, we apply the method that has been demonstrated in the previous section to the evaluation of the cross responses of the system described by Hamiltonian (2). For the response $\hat{\theta}$ of the electric current to the Zeeman interaction with the electromagnetic wave, Eq. (16) rewritten in the coordinate space with allowance for impurities has the form

$$\theta_{ij}(\omega) = \left(\frac{e g \mu_B}{4\pi}\right) \int_{\mathbf{r}_1 - \mathbf{r}_2} v^i_{\delta\alpha}(\mathbf{r}_1) \Pi_{\alpha\beta,\gamma\delta}(\mathbf{r}_1, \mathbf{r}_2; \epsilon_F, \omega) \sigma^j_{\beta\gamma}, \tag{110}$$

where $\hat{\Pi}$ is defined by the series [Eq. (66)] or, that is the same, by Eq. (68). Just as in Eq. (9), the velocity operator $\mathbf{v}(\mathbf{r}_1)$ is the sum of the scalar and spin components

$$\mathbf{v}(\mathbf{r}_1) = \mathbf{v}^{(sc)}(\mathbf{r}_1) + \mathbf{v}^{(sp)}, \tag{111}$$

where the spin component has the same form as in Eq. (9), $\mathbf{v}^{(sp)}_{\delta\alpha} = \alpha(\mathbf{c} \times \boldsymbol{\sigma})_{\delta\alpha}$, whereas the action of the scalar component $\mathbf{v}^{(sc)}(\mathbf{r}_1)$ on the pair of Green's functions $G^A_{\gamma\delta}(\mathbf{r}_2, \mathbf{r}_1)$ and $G^R_{\alpha\beta}(\mathbf{r}_1, \mathbf{r}_2)$, entering and leaving the velocity vertex $\mathbf{v}(\mathbf{r}_1)$ is defined by the following way:[43]

$$G^A_{\gamma\delta}(\mathbf{r}_2, \mathbf{r}_1) \mathbf{v}^{(sc)}_{\delta\alpha}(\mathbf{r}_1) G^R_{\alpha\beta}(\mathbf{r}_1, \mathbf{r}_2) \overset{def}{=} \lim_{\mathbf{r}'_1 \to \mathbf{r}_1} \delta_{\delta\alpha} \frac{1}{2m}[\vec{\pi}(\mathbf{r}_1) + \vec{\pi}^+(\mathbf{r}'_1)] \times G^R_{\alpha\beta}(\mathbf{r}_1, \mathbf{r}_2) G^A_{\gamma\delta}(\mathbf{r}_2, \mathbf{r}'_1),$$

$$\vec{\pi}(\mathbf{r}) = \frac{\nabla_\mathbf{r}}{i} + \frac{e}{c}\mathbf{A}(\mathbf{r}), \quad \vec{\pi}^+(\mathbf{r}') = -\frac{\nabla_{\mathbf{r}'}}{i} + \frac{e}{c}\mathbf{A}(\mathbf{r}'). \tag{112}$$

As at the evaluation of the spin susceptibility, it is convenient to consider the series [Eq. (66)] as the impurity renormalization of the spin vertex. Then expression (110) transforms into

$$\theta_{ij}(\omega) = \tau N(\epsilon_F)\left(\frac{eg\mu_B}{2}\right)\mathrm{Tr}\{v_i^{(1.l)}\Sigma_j^{(r)}(\omega)\}, \quad (113)$$

where $\Sigma_j^{(r)}(\omega)$ is defined by Eq. (85) and, analogously to Eq. (23),

$$\mathbf{v}_{\gamma\beta}^{(1.l)} = n_{imp}U^2\int_{\mathbf{r}_1-\mathbf{r}_2} G_{\gamma\delta}^A(\mathbf{r}_2,\mathbf{r}_1)\mathbf{v}_{\delta\alpha}(\mathbf{r}_1)G_{\alpha\beta}^R(\mathbf{r}_1,\mathbf{r}_2). \quad (114)$$

Like the zero-magnetic-field case, it is sufficient to find $\mathbf{v}^{(1.l)}$ with the accuracy up to $\alpha\bar{\eta}^2$, $\alpha(\omega\tau)$, and $\alpha(\omega_s\tau)$ terms. As in Sec. II C, we have

$$\mathbf{v}_{(1.l)} = \mathbf{v}_{(1.l)}^{(sc)} + \mathbf{v}_{(1.l)}^{(sp)},$$

$$\mathbf{v}_{(1.l)\gamma\beta}^{(sc)} = n_{imp}U^2\int_{\mathbf{r}_1-\mathbf{r}_2} G_{\gamma\delta}^A(\mathbf{r}_2,\mathbf{r}_1)\mathbf{v}_{\delta\alpha}^{(sc)}(\mathbf{r}_1)G_{\alpha\beta}^R(\mathbf{r}_1,\mathbf{r}_2),$$

$$\mathbf{v}_{(1.l)\gamma\beta}^{(sp)} = \alpha(\mathbf{c}\times\boldsymbol{\sigma})_{\delta\alpha}(\alpha\delta|T(\omega)|\gamma\beta). \quad (115)$$

The expansion for $\mathbf{v}_{(1.l)}^{(sc)}$ can be obtained by expanding the Green's functions in powers of H_{so} and H_Z. It can be shown [see Appendix D] that a nonzero contribution to $\mathbf{v}_{(1.l)}^{(sc)}$ comes from the correction of the first order in H_{so} and has the form

$$\mathbf{v}_{(1.l)}^{(sc)} \cong -\alpha(\mathbf{c}\times\boldsymbol{\sigma})(1+i\omega\tau), \quad (116)$$

which differs from Eq. (42) only by the substitution $\tau_0 \to \tau$. Terms on the order of $\alpha\bar{\eta}^2$ and $\alpha(\omega_s\tau)$ are absent in $\mathbf{v}_{(1.l)}^{(sc)}$ but present in $\mathbf{v}_{(1.l)}^{(sp)}$. Use of Eqs. (88) and (91) yields

$$\mathbf{v}_{(1.l)\gamma\beta}^{(sp)} = \alpha\left\{\left[1+i\omega\tau-\frac{\bar{\eta}^2}{2}\right](\mathbf{c}\times\boldsymbol{\sigma})_{\gamma\beta} - \Omega[\boldsymbol{\sigma}(\mathbf{c}\cdot\mathbf{t})-\mathbf{t}(\mathbf{c}\cdot\boldsymbol{\sigma})]_{\gamma\beta}\right\}. \quad (117)$$

Thus

$$\mathbf{v}^{(1.l)} \cong -\alpha\left\{\frac{\bar{\eta}^2}{2}(\mathbf{c}\times\boldsymbol{\sigma}) + \Omega[\boldsymbol{\sigma}(\mathbf{c}\cdot\mathbf{t})-\mathbf{t}(\mathbf{c}\cdot\boldsymbol{\sigma})]\right\} \quad (118)$$

or, in terms of the spin basis $\{\tau\}$,

$$\mathbf{v}^{(1.l)} \cong -\alpha\frac{\bar{\eta}^2}{2}(\mathbf{c}\times\mathbf{f}_{(1)})\tau_1 - \alpha\left[\frac{\bar{\eta}^2}{2}(\mathbf{c}\times\mathbf{f}_{(2)}) - \Omega(\mathbf{c}\times\mathbf{f}_{(3)})\right]\tau_2 - \alpha\left[\frac{\bar{\eta}^2}{2}(\mathbf{c}\times\mathbf{f}_{(3)}) + \Omega(\mathbf{c}\times\mathbf{f}_{(2)})\right]\tau_3. \quad (119)$$

In the sharp-resonance approximation, the terms proportional to Ω dominate. The Bethe-Salpeter Eq. (85) for the right spin-vertex $\boldsymbol{\Sigma}^{(r)}(\omega)$ can be solved just as the corresponding equation for the left spin-vertex $\boldsymbol{\Sigma}^{(l)}(\omega)$ was solved in Sec. III B. The result is

$$\boldsymbol{\Sigma}^{(r)}(\omega) = \sum_{s,m=1}^{3}\mathbf{f}_{(m)}R_{sm}^{-1}\tau_s. \quad (120)$$

Thus $\boldsymbol{\Sigma}^{(r)}$ differs from $\boldsymbol{\Sigma}^{(l)}$ only by the transposition $R_{ms}\to R_{sm}$ that is equivalent to the change $\Omega\to-\Omega$ in Eq. (99). Near the pole ζ_+ we have

$$\mathrm{Tr}\{v_i^{(1.l)}\Sigma_j^{(r)}(\omega)\} \cong \frac{-i\alpha\Omega}{\zeta-\zeta_+}\times\{[-(\mathbf{c}\times\mathbf{f}_{(3)})_i f_{(2)j} + (\mathbf{c}\times\mathbf{f}_{(2)})_i f_{(3)j}] + i[(\mathbf{c}\times\mathbf{f}_{(2)})_i f_{(2)j} + (\mathbf{c}\times\mathbf{f}_{(3)})_i f_{(3)j}]\}. \quad (121)$$

Use of Eq. (108) yields

$$(\mathbf{c}\times\mathbf{f}_{(2)})_i f_{(3)j} - (\mathbf{c}\times\mathbf{f}_{(3)})_i f_{(2)j} = \delta_{ij} - t_i c_j, \quad (122)$$

$$(\mathbf{c}\times\mathbf{f}_{(2)})_i f_{(2)j} + (\mathbf{c}\times\mathbf{f}_{(3)})_i f_{(3)j} = -e_{ijk}c_k - (\mathbf{c}\times\mathbf{t})_i t_j. \quad (123)$$

Finally, at frequencies near the ESR

$$\theta_{ij}(\omega) = N(\epsilon_F)\left(\frac{eg\mu_B}{2}\right)\frac{-\alpha\omega_{res}\tau}{\omega-\omega_{res}+i/T_2}\times\{i[\delta_{ij}(\mathbf{c}\cdot\mathbf{t}) - t_i c_j] + [e_{ijk}c_k + (\mathbf{c}\times\mathbf{t})_i t_j]\}. \quad (124)$$

The evaluation of the kinetic coefficient $\hat{\gamma}$, which describes the response of the spin density to the electric field, can be performed quite analogously. Being written in coordinate representation, Eq. (15) reads

$$\gamma_{ij}(\omega) = \left(\frac{eg\mu_B}{4\pi}\right)\int_{\mathbf{r}_1-\mathbf{r}_2}\sigma_{\delta\alpha}^i\Pi_{\alpha\beta,\gamma\delta}(\mathbf{r}_1,\mathbf{r}_2;\epsilon,\omega)v^j(\mathbf{r}_2)_{\beta\gamma}. \quad (125)$$

As in previous case, we consider impurity insertions as a renormalization of the spin vertex. Then Eq. (125) can be written in the form

$$\gamma_{ij}(\omega) = \tau N(\epsilon_F)\left(\frac{eg\mu_B}{2}\right)\mathrm{Tr}\{\Sigma_i^{(l)}(\omega)v_j^{(1.r)}\}, \quad (126)$$

where, as distinct from Eq. (114), the right velocity-vertex $\mathbf{v}^{(1.r)}$ is defined by

$$\mathbf{v}_{\alpha\delta}^{(1.r)} = n_{imp}U^2\int_{\mathbf{r}_1-\mathbf{r}_2}G_{\alpha\beta}^R(\mathbf{r}_1,\mathbf{r}_2)\mathbf{v}_{\beta\gamma}(\mathbf{r}_2)G_{\gamma\delta}^A(\mathbf{r}_2,\mathbf{r}_1). \quad (127)$$

By repeating the previous analysis, one can show that $\mathbf{v}^{(1.r)}$ differs from $\mathbf{v}^{(1.l)}$ only by the change $\Omega\to-\Omega$, i.e.,

$$\mathbf{v}_i^{(1.r)} \cong -\alpha\frac{\bar{\eta}^2}{2}(\mathbf{c}\times\mathbf{f}_{(1)})^i\tau_1 - \alpha\left[\frac{\bar{\eta}^2}{2}(\mathbf{c}\times\mathbf{f}_{(2)})^i + \Omega(\mathbf{c}\times\mathbf{f}_{(3)})^i\right]\tau_2 - \alpha\left[\frac{\bar{\eta}^2}{2}(\mathbf{c}\times\mathbf{f}_{(3)})^i - \Omega(\mathbf{c}\times\mathbf{f}_{(2)})^i\right]\tau_3. \quad (128)$$

Thus near the ESR and in the sharp-resonance approximation we have

$$\gamma_{ij}(\omega) = N(\epsilon_F)\left(\frac{eg\mu_B}{2}\right)\frac{-\alpha\omega_{res}\tau}{\omega-\omega_{res}+i/T_2} \times\{i[c_it_j-\delta_{ij}(\mathbf{c}\cdot\mathbf{t})]-[e_{ijk}c_k - t_i(\mathbf{c}\times\mathbf{t})_j]\}. \quad (129)$$

By comparing Eqs. (124) and (129), we see that the resonant part of the tensor $\hat{\gamma}$ is the transpose of the resonant part of the tensor $\hat{\theta}$. So the symmetry property [Eq. (49)] apparently is always true. Note that the linear response of the spin polarization to ac electric field in the particular case of in-plane magnetic fields, when the cyclotron motion is suppressed, was also calculated in Ref. 29. Equation (129) agrees with results of Ref. 29 if conditions of applicability of the present and that approach are simultaneously satisfied.

According to Eq. (54), the magnetoelectric contribution to the power loss is determined by the expression

$$\theta_{ij}(\omega) + \gamma_{ij}^{+}(\omega) = -2\alpha\left(\frac{eg\mu_B}{2}\right)N(\epsilon_F)\frac{\omega_{res}(\omega-\omega_{res})\tau}{(\omega-\omega_{res})^2+T_2^{-2}} \times\{i[\delta_{ij}(\mathbf{c}\cdot\mathbf{t})-t_ic_j]+[e_{ijk}c_k+(\mathbf{c}\times\mathbf{t})_it_j]\}. \quad (130)$$

In the case of a free plane wave, when the products $E_i^*B_j$ are real quantities and $\mathbf{E}^*\cdot\mathbf{B}=0$, the magnetoelectric contribution reduces to the form

$$R_{EB} = -\omega\alpha\left(\frac{eg\mu_B}{2}\right)N(\epsilon_F)\frac{\omega_{res}(\omega-\omega_{res})\tau}{(\omega-\omega_{res})^2+T_2^{-2}}(\mathbf{E}_\omega^*\cdot\mathbf{t})(\mathbf{c}\cdot\mathbf{B}_\omega). \quad (131)$$

It can be finite for non-normal incidence. It is interesting to note that since $(\mathbf{E}_\omega^*\cdot\mathbf{t})(\mathbf{c}\cdot\mathbf{B}_\omega)\approx(\mathbf{E}_\omega^*\cdot\mathbf{h})(\mathbf{c}\cdot\mathbf{B}_\omega)$ the magnetoelectric contribution is an odd function of the magnetic field direction $\mathbf{h}$. Although the susceptibility tensor χ_{ij} [as well as the conductivity tensor σ_{ij} of Eq. (141)] also contains the $\mathbf{h}$-odd term $e_{ijk}t_k\approx e_{ijk}h_k$, the corresponding contributions to the absorption vanish when the incident radiation is linearly polarized. In this case, the change in the absorption at the magnetic field reversal is only due R_{EB}. The value of the change is proportional to the value of the constant α of the BSOC. Thus the detection of the difference in the power loss at two opposite directions of the magnetic field provides a possibility for a direct measurement of this constant. The situation is similar to other two known examples of a change in a polar crystal property at the external magnetic field reversal: (i) a change in the energy of an exciton with the momentum $\mathbf{q}$ of the form $\mathbf{q}\cdot(\mathbf{c}\times\mathbf{B}_{(0)})$ (Ref. 44) and (ii) a change in the line width of the ESR on free carriers of the same form when the resonance is excited by inelastic light scattering;[11] $\mathbf{q}$ is then the momentum that is transformed to or taken from the electron system. Note that because of the presence of the scalar product $(\mathbf{c}\cdot\mathbf{B}_\omega)$ in R_{EB} the case of the s polarization of the incident radiation should be excluded.

V. CONDUCTIVITY

In this section, a contribution of the ESR to the electrical conductivity is considered. The fact that the ESR can manifest itself in the conductivity tensor is a direct consequence of the presence of the spin component in the velocity operator. Owing to this circumstance, any collective spin mode can contribute to the correlation function of two velocity operators, which, as is known, is the microscopic expression for the conductivity. By writing Eq. (17) in the coordinate space, we have for the conductivity tensor the expression

$$\sigma_{ij}(\omega) = \left(\frac{e^2}{2\pi}\right)\int_{\mathbf{r}_1-\mathbf{r}_2} v^i_{\delta\alpha}(\mathbf{r}_1)\Pi_{\alpha\beta,\gamma\delta}(\mathbf{r}_1,\mathbf{r}_2;\epsilon_F,\omega)v^j_{\beta\gamma}(\mathbf{r}_2). \quad (132)$$

It is easy to check that the sharp resonance $\sim(\omega-\omega_{res}+i/T_2)^{-1}$ appears in $\sigma_{ij}(\omega)$ only as a result of summation of the infinite series [Eq. (66)]. Therefore, near the resonance, one may safely drop the first term of the series,

$$\left(\frac{e^2}{2\pi}\right)\int_{\mathbf{r}_1-\mathbf{r}_2} v^i_{\delta\alpha}(\mathbf{r}_1)\Pi^{(0)}_{\alpha\beta,\gamma\delta}(\mathbf{r}_1,\mathbf{r}_2;\epsilon_F,\omega)v^j_{\beta\gamma}(\mathbf{r}_2), \quad (133)$$

which has not a pole in the ω plane in a vicinity on the order of T_2^{-1} of the real axis. Then the sum of all other terms

$$\left(\frac{e^2}{2\pi}\right)\int_{\mathbf{r}_1-\mathbf{r}_2} v^i_{\delta\alpha}(\mathbf{r}_1)\left[\int_{\mathbf{r}}\Pi^{(0)}_{\alpha\beta,\gamma\delta}(\mathbf{r}_1,\mathbf{r})n_{imp}U^2 \times\Pi^{(0)}_{\alpha\beta,\gamma\delta}(\mathbf{r},\mathbf{r}_2)+\cdots\right]v^j_{\sigma\tau}(\mathbf{r}_2) \quad (134)$$

takes the form

$$\tau N(\epsilon_F)e^2\{v^{(1.l)}_{i,\alpha\delta}v^{(1.r)}_{j,\delta\alpha}+v^{(1.l)}_{i,\delta\alpha}T_{\alpha\beta,\gamma\delta}(\omega)v^{(1.r)}_{j,\beta\gamma}+\cdots\}, \quad (135)$$

where $\mathbf{v}^{(1.l)}$ and $\mathbf{v}^{(1.r)}$ are defined by Eqs. (114) and (127). If one defines the right renormalized velocity vertex by the equation

$$\mathbf{V}^{(r)}_{\alpha\delta}(\omega) = \mathbf{v}^{(1)}_{\alpha\delta} + T_{\alpha\beta,\gamma\delta}(\omega)\mathbf{V}^{(r)}_{\beta\gamma}(\omega), \quad (136)$$

which is an analog of Eq. (85), the resonant part of the conductivity can be written as

$$\sigma^{(res)}_{ij}(\omega) = \tau N(\epsilon_F)e^2\,\mathrm{Tr}\{v_i^{(1.l)}V_j^{(r)}(\omega)\}. \quad (137)$$

The quantities $\mathbf{v}^{(1.l)}$ and $\mathbf{v}^{(1.r)}$ were found in Sec. IV. Because in the sharp-resonance approximation

$$\mathbf{v}^{(1.r)} \cong \alpha\Omega[\boldsymbol{\sigma}(\mathbf{c}\cdot\mathbf{t})-\mathbf{t}(\mathbf{c}\cdot\boldsymbol{\sigma})], \quad (138)$$

we have

$$\mathbf{V}^{(r)}(\omega) \cong \alpha\Omega[\boldsymbol{\Sigma}^{(r)}(\mathbf{c}\cdot\mathbf{t})-\mathbf{t}(\mathbf{c}\cdot\boldsymbol{\Sigma}^{(r)})]. \quad (139)$$

Use of Eqs. (119) and (120) yields

$$\mathrm{Tr}\{v_i^{(1.l)}V_j^{(r)}(\omega)\} \cong \frac{i(\alpha\Omega)^2}{\zeta-\zeta_+}\{-[(\mathbf{c}\times\mathbf{f}_{(2)})_i(\mathbf{c}\times\mathbf{f}_{(2)})_j+(\mathbf{c}\times\mathbf{f}_{(3)})_i(\mathbf{c}\times\mathbf{f}_{(3)})_j] + i[(\mathbf{c}\times\mathbf{f}_{(2)})_i(\mathbf{c}\times\mathbf{f}_{(3)})_j-(\mathbf{c}\times\mathbf{f}_{(3)})_i(\mathbf{c}\times\mathbf{f}_{(2)})_j]\}. \quad (140)$$

Finally, by making use of Eq. (108), we obtain

$$\sigma_{ij}^{res}(\omega) \cong e^2N(\epsilon_F)\frac{i(\alpha\omega_{res}\tau)^2}{\omega-\omega_{res}+i/T_2} \times[c_ic_j-\delta_{ij}+(\mathbf{c}\times\mathbf{t})_i(\mathbf{c}\times\mathbf{t})_j+ie_{ijs}c_s(\mathbf{c}\cdot\mathbf{t})]. \quad (141)$$

Compare the power loss due to the electric component of the microwave field, $\frac{1}{2}(\mathbf{E}_\omega^*\cdot\hat{\sigma}'(\omega)\cdot\mathbf{E}_\omega)$, and due to magnetic component, $\frac{\omega}{2}(\mathbf{B}_\omega^*\cdot\hat{\chi}''(\omega)\cdot\mathbf{B}_\omega)$. First, Eqs. (109) and (141) show that the polarization dependence of the electric and magnetic terms coincides only at the perpendicular external magnetic field $\mathbf{B}_{(0)}\|\mathbf{c}$. For the ratio of their absolute values we have

$$\frac{R_B}{R_E}\sim\frac{\omega_{res}\chi''}{\sigma'}\sim\left[\left(\frac{g}{\epsilon_F\tau}\right)\frac{(m/m_0)(v_F/c)}{\delta}\right]^2, \quad (142)$$

where m_0 is the electron mass in vacuum and $\delta=\alpha p_F/\epsilon_F$. The value of the BSOC enters the denominator of this expression through the parameter δ, which is very small. For the $In_xGa_{1-x}As/In_yAl_{1-y}As$ heterostructure, which may be considered as a typical example, $m/m_0=0.046$, $\alpha=1.4\times10^{-10}$ eV so that the spin-orbit splitting $2\alpha p_F$ at the electron density $n_s\simeq1\times10^{12}$ cm^{-2} is equal to 0.65 meV. Since the Fermi energy $\epsilon_F\simeq40$ meV, we have $\delta\simeq10^{-3}$. This smallness, however, is compensated by two small parameters entering the numerator: $m/m_0\simeq10^{-2}$ and $v_F/c\simeq10^{-3}$ so that the total value of the ratio can be rather small. Thus the electric component of the resonant microwave field can excite the resonance much more effectively than the magnetic component in spite of a small value of the spin-orbit coupling; an analogous fact was earlier known for donor-bound electrons in bulk crystals.[16] This agrees with results of recent experiments performed on A_3B_5 quantum well,[4] where the ESR was detected only when the 2D structure was placed in an antinode of electric component of the microwave field.

VI. SUMMARY

In this paper, the method for analyzing spin-dependent kinetic problems in conducting media with the band spin-orbit coupling has been suggested. The key statement of the method, in the Feynman-diagram language, is that by summing *infinite* series of ladder kinetic diagrams responsible for or contributing to a physical phenomenon, it is sufficient by evaluating every *single* rung of the ladder to treat the BSOC (and the Zeeman interaction) by means of the perturbation theory. This approach has been applied to the ESR on conduction electrons in an impure asymmetric two-dimensional semiconductor structure at tilted magnetic fields. Predictions for the dependence of the resonant absorption upon the direction of the magnetic field and the polarization of the incident radiation have been made that are subject to experimental verification. Two features of the ESR have been revealed: (i) the electric component of the microwave field can excite the ESR more effectively than the magnetic component. (ii) Due to the BSOC, the cyclotron motion contributes to the spin precession; in particular, the ESR can exist even if the g factor of current carriers equals zero, i.e., in the absence of the Zeeman interaction. In this case, the ESR is maintained solely by the cyclotron motion. The constitutive relations distinctive of the macroscopic electrodynamics of broken-mirror-symmetry conducting media have been formulated as well.

The method is applicable also to bulk crystals and could serve as a general framework for analyzing both linear and nonlinear responses of the system to external electromagnetic perturbations. In particular, it would be interesting to consider the ESR in a bulk polar semiconductor. In that case, the relationship between electric, magnetic, and magneto-electric contributions to the absorption could be different because of the suppression of the electric component of the microwave field with respect to the magnetic one within the skin layer. For the case of 2D structures in which both the Rashba type and Dresselhaus type of spin-orbit coupling are present and/or with an anisotropic g factor, an extension of the method offers no principal difficulty. It would be also interesting to find out the influence of interparticle collisions on spin relaxation which should increase with the grow of temperature. A more difficult problem is to evaluate the ESR in the quantum Hall regime when the interparticle interaction plays a substantial role.

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APPENDIX A

In this appendix, we present some details of the derivation of Eqs. (33) and (42). In the absence of an external magnetic field, the Green's function of the Hamiltonian H_0 averaged over impurities has the form

$$G_{\kappa\beta}^{R(A)(0)}(\epsilon_F+\omega,\mathbf{p})=g^{R(A)}(\omega,\xi)\delta_{\kappa\beta},$$

$$g^{R(A)}(\omega,\xi)=\left(\omega-\xi\pm\frac{1}{2\tau_0}\right)^{-1}, \quad (A1)$$

where $\xi=\frac{p^2}{2m}-\epsilon_F$. Equation (33) readily follow from the equalities

$$\int\frac{d\xi}{2\pi\tau_0}\begin{pmatrix} g^R(\omega,\xi)g^A(0,\xi) \\ [g^R(0,\xi)g^A(0,\xi)]^2 \\ [g^R(0,\xi)]^3g^A(0,\xi) \\ g^R(0,\xi)[g^A(0,\xi)]^3 \end{pmatrix}=\begin{pmatrix} (1-i\omega\tau_0)^{-1} \\ 2\tau_0^2 \\ -\tau_0^2 \\ -\tau_0^2 \end{pmatrix}. \quad (A2)$$

Because $p_{(0.0)}$, $p_{(2.0)}$, and $p_{(0.2)}$ are proportional to the small parameter η^2, we have dropped a dependence of these quantities on the small parameter $\omega\tau_0$.

Consider the velocity-vertex $\mathbf{v}^{(1)}(\omega)$ given by Eqs. (36)–(38) From Eqs. (32) and (40) we have

$$\mathbf{v}^{(sc)}_{(1.1)\alpha\beta}(\omega) = \int \frac{d\hat{p}}{2\pi}\int \frac{d\xi}{2\pi\tau_0}\frac{\mathbf{p}}{m}\alpha(\mathbf{p}\times\mathbf{c}\cdot\boldsymbol{\sigma})_{\alpha\beta}\{[g^R(\omega,\xi)]^2 g^A(0,\xi) + g^R(\omega,\xi)[g^A(0,\xi)]^2\}$$
$$= \alpha(\mathbf{c}\times\boldsymbol{\sigma})_{\alpha\beta}\int \frac{d\xi}{2\pi\tau_0}\left(\frac{p^2}{2m}\right)\{[g^R(\omega,\xi)]^2 g^A(0,\xi) + g^R(\omega,\xi)[g^A(0,\xi)]^2\}. \tag{A3}$$

Use of the equalities

$$\int \frac{d\xi}{2\pi\tau_0}(\epsilon_F+\xi)\begin{pmatrix}[g^R(\omega,\xi)]^2 g^A(0,\xi)\\ g^R(\omega,\xi)[g^A(0,\xi)]^2\end{pmatrix} = \frac{i\tau_0}{(1-i\omega\tau_0)^2}\begin{pmatrix}-\epsilon_F+\dfrac{i}{2\tau_0}\\ \epsilon_F+\omega+\dfrac{i}{2\tau_0}\end{pmatrix} \tag{A4}$$

reduces Eq. (A3) to Eq. (42). From Eqs. (32) and (41) we have

$$\mathbf{v}^{(sc)}_{(1.3)\alpha\beta}(\omega) = \int \frac{d\hat{p}}{2\pi}\int \frac{d\xi}{2\pi\tau_0}\frac{\mathbf{p}}{m}\alpha^3 p^2(\mathbf{p}\times\mathbf{c}\cdot\boldsymbol{\sigma})_{\alpha\beta}$$
$$\times\{[g^R(\omega,\xi)]^4 g^A(0,\xi) + [g^R(\omega,\xi)]^3[g^A(0,\xi)]^2 + [g^R(\omega,\xi)]^2[g^A(0,\xi)]^3 + g^R(\omega,\xi)[g^A(0,\xi)]^4\}$$
$$= \alpha^3 2m(\mathbf{c}\times\boldsymbol{\sigma})_{\alpha\beta}\int \frac{d\xi}{2\pi\tau_0}(\epsilon_F+\xi)^2$$
$$\times\{[g^R(\omega,\xi)]^4 g^A(0,\xi) + [g^R(\omega,\xi)]^3[g^A(0,\xi)]^2 + [g^R(\omega,\xi)]^2[g^A(0,\xi)]^3 + g^R(\omega,\xi)[g^A(0,\xi)]^4\}. \tag{A5}$$

Use of the equalities

$$\int \frac{d\xi}{2\pi\tau_0}(\epsilon_F+\xi)^2\begin{pmatrix}[g^R(\omega,\xi)]^4 g^A(0,\xi)\\ [g^R(\omega,\xi)]^3[g^A(0,\xi)]^2\\ [g^R(\omega,\xi)]^2[g^A(0,\xi)]^3\\ g^R(\omega,\xi)[g^A(0,\xi)]^4\end{pmatrix} = \frac{i\tau_0^3}{(1-i\omega\tau_0)^4}\begin{pmatrix}\left(\epsilon_F-\dfrac{i}{2\tau_0}\right)^2\\ \left(\epsilon_F-\dfrac{i}{2\tau_0}\right)\left[-3\left(\epsilon_F-\dfrac{i}{2\tau_0}\right)-\dfrac{2i}{\tau_0}(1-i\omega\tau_0)\right]\\ \left(\epsilon_F+\omega+\dfrac{i}{2\tau_0}\right)\left[3\left(\epsilon_F+\omega+\dfrac{i}{2\tau_0}\right)-\dfrac{2i}{\tau}(1-i\omega\tau_0)\right]\\ -\left(\epsilon_F+\omega+\dfrac{i}{2\tau_0}\right)^2\end{pmatrix} \tag{A6}$$

yields $\mathbf{v}^{(sc)}_{(1.3)}=0$.

APPENDIX B

In this appendix, we show how one can derive equations of Sec. III A. Because the technique of summing over Landau levels is presented in Ref. 22 in tiny details, we do not reproduce it here. Only final results for sums encountered are given. The same also refers to Appendix D.

(1) Consider the Green's function $G^{R(A)}_{(0)\alpha\beta}(\mathbf{r}_1,\mathbf{r}_2;\epsilon)$ given by Eq. (72). If one introduces a complex notation for the position vector, with

$$z = \frac{1}{\lambda}(x+iy), \quad z^* = \frac{1}{\lambda}(x-iy), \tag{B1}$$

the eigenstates of H_0 can be written in the form[21]

$$\langle \mathbf{r}|n,l\rangle \equiv \varphi_{nl}(z,z^*) = (2\pi\lambda^2 2^{n+l}n!l!)^{-1/2}\tilde{\varphi}_{nl}(z,z^*), \tag{B2}$$

$$\tilde{\varphi}_{nl}(z,z^*) = e^{zz^*}(2\partial_z)^l(2\partial_{z^*})^n e^{-zz^*/2}, \quad n,l=0,1,\ldots \tag{B3}$$

Use of the representation

$$\tilde{\varphi}_{nl}(z,z^*) = \lim_{\lambda=\mu=0}(\partial_\lambda)^l(\partial_\mu)^n e^{-2\lambda\mu}e^{-\mu z-\lambda z^*}e^{-zz^*/4} \tag{B4}$$

yields

$$\sum_l \langle \mathbf{r}_1|n,l\rangle\langle n,l|\mathbf{r}_2\rangle = \exp\left[\frac{1}{4}(z_1^* z_2 - z_1 z_2^*)\right]g_n(z_1-z_2),$$

$$g_n(z) = \frac{1}{2\pi\lambda^2}e^{-|z|^2/4}L_n\left(\frac{|z|^2}{2}\right), \tag{B5}$$

where L_n denotes the Laguerre polynomial.[45] We adopt that the unit vector $\mathbf{h}_\perp=\mathbf{B}^{(0)}_\perp/|\mathbf{B}^{(0)}_\perp|$ (directed perpendicular to the plane of the electron structure) and unit vectors $\hat{x}$, $\hat{y}$ (lying in the plane of the electron structure) form the right-handoriented basis. Then, by making use of Eq. (B5) and the equality

$$z_1^* z_2 - z_1 z_2^* = 2i\mathbf{h}_\perp \cdot (\mathbf{r}_1 \times \mathbf{r}_2), \tag{B6}$$

one can represent $\hat{G}_{(0)}$ in the form

$$G^{R(A)}_{(0)\alpha\beta}(\mathbf{r}_1,\mathbf{r}_2;\epsilon) = \delta_{\alpha\beta} \exp\left[\frac{i}{2\lambda^2}\mathbf{h}_\perp \cdot (\mathbf{r}_1 \times \mathbf{r}_2)\right] \sum_n g_n(\mathbf{r}_1 - \mathbf{r}_2) \times G_n^{R(A)}(\epsilon), \tag{B7}$$

where

$$g_n(\mathbf{r}) \overset{def}{=} g_n(z). \tag{B8}$$

From Eqs. (75) and (B7) we find

$$\langle \alpha\delta | T^{(0)}(\omega) | \gamma\beta \rangle = \delta_{\alpha\beta}\delta_{\gamma\delta} n_{imp} |U|^2 \int_{\mathbf{r}_1 - \mathbf{r}_2} \sum_{n,m} g_m(z_2 - z_1) \times g_n(z_1 - z_2) G_m^A(\epsilon_F) G_n^R(\epsilon_F + \omega). \tag{B9}$$

Taking account of the orthogonality of the g_n functions

$$\int d^2 r g_n(\mathbf{r}) g_m(-\mathbf{r}) = \Delta_{n,m}(2\pi\lambda^2)^{-1} \tag{B10}$$

and the equality

$$\sum_n G_n^A G_n^R = \frac{1}{\omega + i/\tau} \frac{2\pi i N(\epsilon_F)}{p}, \tag{B11}$$

where $p = (2\pi\lambda^2)^{-1}$ is the degeneracy or the number of states per Landau level, $N(\epsilon)$ is the density of states connected with $\tau(\epsilon)$ by the relation (69), $G_n^R \equiv G_n^R(\epsilon_F + \omega)$, and $G_n^A \equiv G_n^A(\epsilon_F)$, we come to Eq. (73).

(2) Substituting Eq. (72) into Eq. (74), we have

$$G^{R(A)}_{(1.Z)}(\mathbf{r}_1,\mathbf{r}_2;\epsilon)_{\alpha\beta} = \frac{\omega_s}{2}(\mathbf{h}\cdot\sigma)_{\alpha\beta} \sum_{n,l} \langle \mathbf{r}_1 | n,l \rangle \langle n,l | \mathbf{r}_2 \rangle [G_n^{R(A)}]^2, \tag{B12}$$

where $\mathbf{h} = \mathbf{B}_{(0)}/|\mathbf{B}_{(0)}|$. Then it follows from Eqs. (75) and (B12) that

$$\langle \alpha\delta | T^{(1)}_{par} | \gamma\beta \rangle = n_{imp}|U|^2 \frac{\omega_s}{2} p \sum_n \{(\mathbf{h}\cdot\boldsymbol{\sigma})_{\alpha\beta}\delta_{\gamma\delta}[G_n^R]^2 G_n^A + \delta_{\alpha\beta}(\mathbf{h}\cdot\boldsymbol{\sigma})_{\gamma\delta} G_n^R [G_n^A]^2\}. \tag{B13}$$

Use of the equalities

$$\sum_n [G_n^R]^2 G_n^A = -\tau^2 \frac{2\pi i}{p} N(\epsilon_F),$$

$$\sum_n G_n^R [G_n^A]^2 = \tau^2 \frac{2\pi i}{p} N(\epsilon_F) \tag{B14}$$

transforms Eq. (B13) to Eq. (76).

(3) Consider corrections due to H_{so}. First note that H_{so} can be written in the form

$$H_{so} = -\frac{\alpha}{\lambda}(\mathbf{h}_\perp \cdot \mathbf{c})\left[\hat{s}_-\left(2\partial^* - \frac{z}{2}\right) - \hat{s}_+\left(2\,\partial + \frac{z^*}{2}\right)\right], \tag{B15}$$

where $\hat{s}_\pm = (\hat{\sigma}_x \pm \hat{\sigma}_y)/2$. It follows from Eqs. (B2)–(B4) that

$$\left(2\partial^* - \frac{z}{2}\right)\varphi_{n,l} = \sqrt{2(n+1)}\varphi_{n+1,l}, \tag{B16}$$

$$\left(2\,\partial + \frac{z^*}{2}\right)\varphi_{n,l} = -\sqrt{2n}\varphi_{n-1,l}. \tag{B17}$$

With the help of these equations and the integration-by-part rules

$$\int dz dz^* g^*(z) \begin{pmatrix} \partial \\ z \\ \partial^* \\ z^* \end{pmatrix} f(z) = \int dz dz^* \left[\begin{pmatrix} -\partial^* \\ z^* \\ -\partial \\ z \end{pmatrix} g(z)\right]^* f(z), \tag{B18}$$

which are valid for any differentiable functions $f(z)$ and $g(z)$ sufficiently rapidly vanishing at infinity, one can check that

$$\int_{\mathbf{r}} \langle n_1 l_1 | \mathbf{r} \rangle H_{so}(\mathbf{r}) \langle \mathbf{r} | n_2 l_2 \rangle = -\frac{\alpha}{\lambda}(\mathbf{h}_\perp \cdot \mathbf{c})\Delta_{l_1 l_2}[\hat{s}_-\sqrt{2(n_2+1)}\Delta_{n_1,n_2+1} + \hat{s}_+\sqrt{2(n_1+1)}\Delta_{n_1+1,n_2}]. \tag{B19}$$

After substituting Eq. (B19) into Eq. (77), we find

$$\hat{G}^{R(A)}_{(1.so)}(\mathbf{r}_1,\mathbf{r}_2) = -\frac{\alpha}{\lambda}(\mathbf{h}_\perp \cdot \mathbf{c}) \sum_{n,l} \sqrt{2(n+1)} \times G_n^{R(A)} G_{n+1}^{R(A)} [\hat{s}_- \langle \mathbf{r}_1 | n+1,l \rangle \langle n,l | \mathbf{r}_2 \rangle + \hat{s}_+ \langle \mathbf{r}_1 | n,l \rangle \langle n+1,l | \mathbf{r}_2 \rangle]. \tag{B20}$$

By making use of the representation [Eqs. (B2)–(B4)], one can show that

$$\sum_l \begin{pmatrix} \langle \mathbf{r}_1 | n,l \rangle \langle n+1,l | \mathbf{r}_2 \rangle \\ \langle \mathbf{r}_1 | n+1,l \rangle \langle n,l | \mathbf{r}_2 \rangle \end{pmatrix} = \exp\left[\frac{1}{4}(z_1^* z_2 - z_1 z_2^*)\right] \begin{pmatrix} g_n^{(1a)}(z_1 - z_2) \\ g_n^{(1b)}(z_1 - z_2) \end{pmatrix}, \tag{B21}$$

where

$$\begin{pmatrix} g_n^{(1a)}(z) \\ g_n^{(1b)}(z) \end{pmatrix} = \frac{e^{-|z|^2/4}}{2\pi\lambda^2\sqrt{2(n+1)}} L_n^1\left(\frac{|z|^2}{2}\right)\begin{pmatrix} z^* \\ -z \end{pmatrix} \tag{B22}$$

and L_n^k denotes the associate Laguerre polynomial.[45] Thus

$$\hat{G}^{R(A)}_{(1.so)}(\mathbf{r}_1,\mathbf{r}_2) = -\frac{\alpha}{\lambda}(\mathbf{h}_\perp\cdot\mathbf{c})\sum_n \sqrt{2(n+1)}G_n^{R(A)}G_{n+1}^{R(A)} \times \exp\left[\frac{1}{4}(z_1^* z_2 - z_1 z_2^*)\right][\hat{s}_- g_n^{(1b)}(z_1-z_2) + \hat{s}_+ g_n^{(1a)}(z_1-z_2)]. \quad (B23)$$

The first-order correction $\hat{T}^{(1)}_{so}$ given by Eq. (78) vanishes because of the orthogonality of g_n and $g_n^{(1a,b)}$ functions

$$\int_{\mathbf{r}} g_m(\mathbf{r})g_n^{(1a)}(-\mathbf{r}) = \int_{\mathbf{r}} g_m(\mathbf{r})g_n^{(1b)}(-\mathbf{r}) = 0. \quad (B24)$$

(4) Equation (79), with the help of Eq. (B19), can be transformed to the form

$$\hat{G}^{R(A)}_{(2.so)}(\mathbf{r}_1,\mathbf{r}_2,\epsilon_F) = 2\left(\frac{\alpha}{\lambda}\right)^2 \sum_n (n+1)\{G^{R(A)}_{n+1}[G_n^{R(A)}]^2\langle\mathbf{r}_1|n,l\rangle\langle n,l|\mathbf{r}_2\rangle\hat{\Pi}^{(u)} + [G^{R(A)}_{n+1}]^2 G_n^{R(A)}\langle\mathbf{r}_1|n+1,l\rangle\langle n+1,l|\mathbf{r}_2\rangle\hat{\Pi}^{(d)}\}, \quad (B25)$$

where $\hat{\Pi}^{(u,d)} = \frac{1}{2}(1 \pm \mathbf{h}_\perp\cdot\boldsymbol{\sigma})$. Now, by making use of Eqs. (B23) and (B25), for the quantities $\hat{P}_{(1.1)}$, $\hat{P}_{(2.0)}$, and $\hat{P}_{(0.2)}$ defined by Eq. (80), one can obtain the following expressions:

$$\langle\alpha\delta|P_{(1.1)}|\gamma\beta\rangle = n_{imp}|U|^2\int_{\mathbf{r}_1-\mathbf{r}_2}\left(\frac{\alpha}{\lambda}\right)^2\sum_{n,m}\sqrt{2(m+1)}G^A_{m+1}G^A_m\sqrt{2(n+1)}G^R_{n+1}G^R_n \times [g_m^{(1a)}(\mathbf{r}_2-\mathbf{r}_1)\hat{s}_+ + g_m^{(1b)}(\mathbf{r}_2-\mathbf{r}_1)\hat{s}_-]_{\gamma\delta}[g_n^{(1a)}(\mathbf{r}_1-\mathbf{r}_2)\hat{s}_+ + g_n^{(1b)}(\mathbf{r}_1-\mathbf{r}_2)\hat{s}_-]_{\alpha\beta}, \quad (B26)$$

$$\langle\alpha\delta|P_{(2.0)}|\gamma\beta\rangle = n_{imp}|U|^2\int_{\mathbf{r}_1-\mathbf{r}_2} 2\left(\frac{\alpha}{\lambda}\right)^2\sum_{n,m}(n+1)G^A_m g_m(r_2-r_1)\delta_{\gamma\delta} \times [G^R_{n+1}(G^R_n)^2 g_n(\mathbf{r}_1-\mathbf{r}_2)\hat{\Pi}^{(u)}_{\alpha\beta} + (G^R_{n+1})^2 G^R_n g_{n+1}(\mathbf{r}_1-\mathbf{r}_2)\hat{\Pi}^{(d)}_{\alpha\beta}], \quad (B27)$$

$$\langle\alpha\delta|P_{(0.2)}|\gamma\beta\rangle = n_{imp}|U|^2\int_{\mathbf{r}_1-\mathbf{r}_2} 2\left(\frac{\alpha}{\lambda}\right)^2\sum_{n,m}(m+1)G^R_n g_n(\mathbf{r}_1-\mathbf{r}_2)\delta_{\alpha\beta} \times [G^A_{m+1}(G^A_m)^2 g_m(\mathbf{r}_2-\mathbf{r}_1)\hat{\Pi}^{(u)}_{\gamma\delta} + (G^A_{m+1})^2 G^A_m g_{m+1}(\mathbf{r}_2-\mathbf{r}_1)\hat{\Pi}^{(d)}_{\gamma\delta}]. \quad (B28)$$

By making use of the orthogonality property

$$\int_{\mathbf{r}}\begin{pmatrix} g_n^{(1a)}(\mathbf{r})g_m^{(1a)}(-\mathbf{r}) & g_n^{(1a)}(\mathbf{r})g_m^{(1b)}(-\mathbf{r}) \\ g_n^{(1b)}(\mathbf{r})g_m^{(1a)}(-\mathbf{r}) & g_n^{(1b)}(\mathbf{r})g_m^{(1b)}(-\mathbf{r})\end{pmatrix} = \Delta_{n,m}\frac{1}{2\pi\lambda^2}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}, \quad (B29)$$

together with Eqs. (B10) and (B24), one can reduce expressions (B26)–(B28) to the form

$$\langle\alpha\delta|P_{(1.1)}|\gamma\beta\rangle = n_{imp}|U|^2\frac{\alpha^2}{\pi\lambda^3}\sum_n (n+1)G^R_{n+1}G^R_n G^A_{n+1}G^A_n \times (\hat{s}^+_{\gamma\delta}\hat{s}^-_{\alpha\beta} + \hat{s}^-_{\gamma\delta}\hat{s}^+_{\alpha\beta}), \quad (B30)$$

$$\langle\alpha\delta|P_{(2.0)}|\gamma\beta\rangle = n_{imp}|U|^2\frac{\alpha^2}{\pi\lambda^3}\sum_n (n+1)\delta_{\gamma\delta} \times [G^R_{n+1}(G^R_n)^2 G^A_n\hat{\Pi}^{(u)}_{\alpha\beta} + (G^R_{n+1})^2 G^R_n G^A_{n+1}\hat{\Pi}^{(d)}_{\alpha\beta}], \quad (B31)$$

$$\langle\alpha\delta|P_{(0.2)}|\gamma\beta\rangle = n_{imp}|U|^2\frac{\alpha^2}{\pi\lambda^3}\sum_n (n+1)\delta_{\alpha\beta} \times [G^R_n G^A_{n+1}(G^A_n)^2\hat{\Pi}^{(u)}_{\gamma\delta} + G^R_{n+1}(G^A_{n+1})^2 G^A_n\hat{\Pi}^{(d)}_{\gamma\delta}]. \quad (B32)$$

Now use of the equalities

$$\sum_n \epsilon_n G^R_{n+1}G^R_n G^A_{n+1}G^A_n = \frac{2\tau^3\epsilon_F}{1+(\omega_c\tau)^2}\frac{2\pi}{p}N(\epsilon_F),$$

$$\sum_n \epsilon_n G^R_{n+1}(G^R_n)^2 G^A_n = -\frac{2\tau^3\epsilon_F}{1+i\omega_c\tau}\frac{2\pi}{p}N(\epsilon_F),$$

$$\sum_n \epsilon_n (G^R_{n+1})^2 G^R_n G^A_{n+1} = -\frac{2\tau^3\epsilon_F}{1-i\omega_c\tau}\frac{2\pi}{p}N(\epsilon_F) \quad (B33)$$

reduces Eqs. (B30)–(B32) to Eq. (81).

APPENDIX C

In this appendix, a list of simply verified Fierz-type identities for the tensor products of Pauli matrices used in the main text is presented. These identities are as follows:[20,46]

$$(\mathbf{n}\cdot\boldsymbol{\sigma})_{\alpha\beta}\delta_{\gamma\delta}-\delta_{\alpha\beta}(\mathbf{n}\cdot\boldsymbol{\sigma})_{\gamma\delta}=i\mathbf{n}\cdot(\boldsymbol{\sigma}_{\alpha\delta}\times\boldsymbol{\sigma}_{\gamma\beta}) \equiv ie_{ljk}n^l\sigma^j_{\alpha\delta}\sigma^k_{\gamma\beta}, \quad (C1)$$

$$\delta_{\gamma\delta}\delta_{\alpha\beta}=\frac{1}{2}(\delta_{\alpha\delta}\delta_{\gamma\beta}+\boldsymbol{\sigma}_{\alpha\delta}\cdot\boldsymbol{\sigma}_{\gamma\beta}), \quad (C2)$$

$$(\mathbf{n}\times\boldsymbol{\sigma})_{\gamma\delta}(\mathbf{n}\times\boldsymbol{\sigma})_{\alpha\beta}=\delta_{\alpha\delta}\delta_{\gamma\beta}-(\mathbf{n}\cdot\boldsymbol{\sigma})_{\alpha\delta}(\mathbf{n}\cdot\boldsymbol{\sigma})_{\gamma\beta}, \quad (C3)$$

where $\mathbf{n}$ is any three-dimensional vector.

APPENDIX D

In this appendix, a derivation of Eq. (116) is given. Just as in Sec. II C, the expansion of $\mathbf{v}^{(sc)}_{(1.l)}$ in powers of α and $\omega_s\tau$ is obtained by means of the expansion of the exact Green's functions in series in H_{so} and H_Z. For the same reasons as in the zero-magnetic-field case, only terms with odd numbers of H_{so} are nonzero. With the required accuracy, they are $\mathbf{v}^{(sc)}_{(1.so)}$, which is linear in H_{so}, $\mathbf{v}^{(sc)}_{(2.so,Z)}$, which is bilinear in H_{so} and H_Z, and $\mathbf{v}^{(sc)}_{(3.so)}$, which is of the third order in H_{so}.

The derivation is essentially based on a complex representation of the vector operator $\mathbf{v}^{(sc)}(\mathbf{r})$ introduced in Ref. 21. Namely, for any 2D vector $\mathbf{p}$, it is valid the representation

$$\mathbf{p}=\frac{1}{2}(pe^*+p^*e), \quad (D1)$$

where

$$e=\hat{x}+i\hat{y}, \quad p=p_x+ip_y. \quad (D2)$$

In particular, for the vector $\vec{\pi}(\mathbf{r})$ of Eq. (112) we have

$$\vec{\pi}(\mathbf{r})=\frac{1}{2}(\pi e^*+\pi^* e), \quad (D3)$$

where

$$\pi=-\frac{i}{\lambda}\left(2\partial^*-\frac{z}{2}\right), \quad \pi^*=-\frac{i}{\lambda}\left(2\,\partial+\frac{z^*}{2}\right) \quad (D4)$$

with $\partial\equiv\partial/\partial z$, $\partial^*\equiv\partial/\partial z^*$, and z is defined by Eq. (B1). By making use of Eqs. (B16), (B17), (D3), and (D4), we have

$$\vec{\pi}(\mathbf{r})\langle\mathbf{r}|n,l\rangle=-\frac{i}{2\lambda}[e^*\sqrt{2(n+1)}\langle\mathbf{r}|n+1,l\rangle - e\sqrt{2n}\langle\mathbf{r}|n-1,l\rangle] \quad (D5)$$

and

$$\langle n,l|\mathbf{r}'\rangle\vec{\pi}^+(\mathbf{r}')=\frac{i}{2\lambda}[e\sqrt{2(n+1)}\langle n+1,l|\mathbf{r}'\rangle - e^*\sqrt{2n}\langle n-1,l|\mathbf{r}'\rangle], \quad (D6)$$

where the function $\langle\mathbf{r}|n,l\rangle$ is defined by Eq. (B2).

(1) In this auxiliary item, we consider the contribution of spin-orbit free Green's functions to $\mathbf{v}^{(sc)}_{(1.l)}$ defined by Eq. (115). In the absence of an external magnetic field, the contribution vanishes due to integration of the $\mathbf{p}$-linear expression $\int_{\mathbf{p}}[G^{A(0)}\mathbf{p}G^{R(0)}]$ over all momentum space. We show here that the reason for nullification of

$$\mathbf{v}^{(sc)(0)}_{(1.l)}\overset{def}{=}n_{imp}U^2\int_{\mathbf{r}_1-\mathbf{r}_2}\lim_{\mathbf{r}'_1\to\mathbf{r}_1}G^{A(0)}(\mathbf{r}_2,\mathbf{r}'_1)\frac{1}{2m}[\vec{\pi}^+(\mathbf{r}'_1)+\vec{\pi}(\mathbf{r}_1)] \times G^{R(0)}(\mathbf{r}_1,\mathbf{r}_2) \quad (D7)$$

is of the same geometrical nature. Note that for any sufficiently rapidly convergent spin-matrix functions $F(\mathbf{r})$ and $G(\mathbf{r})$ it holds the equality

$$\int_{\mathbf{r}_1-\mathbf{r}_2}\lim_{\mathbf{r}'_1\to\mathbf{r}_1}F_{\gamma\rho}(\mathbf{r}_2-\mathbf{r}'_1)\vec{\pi}^+(\mathbf{r}'_1)G_{\rho\kappa}(\mathbf{r}_1-\mathbf{r}_2) = \int_{\mathbf{r}_1-\mathbf{r}_2}F_{\gamma\rho}(\mathbf{r}_2-\mathbf{r}_1)\vec{\pi}(\mathbf{r}_1)G_{\rho\kappa}(\mathbf{r}_1-\mathbf{r}_2). \quad (D8)$$

Therefore, it is sufficient to calculate only one of the terms in Eq. (D7), for example, the second one. By making use of Eqs. (B21) and (D5), we have

$$G^{A(0)}(\mathbf{r}_2,\mathbf{r}'_1)\frac{1}{2m}\vec{\pi}(\mathbf{r}_1)G^{R(0)}(\mathbf{r}_1,\mathbf{r}_2) = \frac{-i}{4m\lambda}\sum_{m,n}G^A_m g_m(\mathbf{r}_2-\mathbf{r}_1)\times[g^{(1b)}_n(\mathbf{r}_1-\mathbf{r}_2)e^*\sqrt{2(n+1)} - g^{(1a)}_{n-1}(\mathbf{r}_2-\mathbf{r}_1)e\sqrt{2n}]G^R_n. \quad (D9)$$

Here $g_n(z)$ [see Eq. (B5)] is a function of $|z|^2$, i.e., behaves as a scalar at rotations of the complex plane, whereas functions $g^{(1.a,b)}_n(z)$ [see Eq. (B22)] have the form $zf(|z|^2)$ or $z^*f(|z|^2)$, i.e., behave as vectors. Therefore, the coordinate integration makes Eq. (D9) vanish. Formally, it follows from Eq. (B24). The first term in Eq. (D7) also vanishes in view of Eq. (D8).

(2) Consider a term of the expansion linear in H_{so}. Quite analogously to Eq. (40), we have

$$\mathbf{v}^{(sc)}_{(1.so)}=n_{imp}U^2\int_{\mathbf{r}_1-\mathbf{r}_2}\lim_{\mathbf{r}'_1\to\mathbf{r}_1}\Big\{G^{A(0)}(\mathbf{r}_2,\mathbf{r}'_1)\frac{1}{2m} \times[\vec{\pi}^+(\mathbf{r}'_1)+\vec{\pi}(\mathbf{r}_1)]G^{R(1)}(\mathbf{r}_1,\mathbf{r}_2) + G^{A(1)}(\mathbf{r}_2,\mathbf{r}'_1)\frac{1}{2m}[\vec{\pi}^+(\mathbf{r}'_1)+\vec{\pi}(\mathbf{r}_1)]G^{R(0)}(\mathbf{r}_1,\mathbf{r}_2)\Big\}. \quad (D10)$$

All below, for the sake of brevity, the procedure of splitting $\mathbf{r}_1$ to $\mathbf{r}_1$ and $\mathbf{r}'_1$ with the following taking the limit $\mathbf{r}'_1\to\mathbf{r}_1$ is implicit. Due to Eq. (D8), the right-hand side of Eq. (D10) is equal to

$$2n_{imp}U^2\int_{\mathbf{r}_1-\mathbf{r}_2}\left[G^{A(0)}(\mathbf{r}_2,\mathbf{r}_1)\frac{1}{2m}\vec{\pi}^+(\mathbf{r}_1)G^{R(1)}(\mathbf{r}_1,\mathbf{r}_2)+G^{A(1)}(\mathbf{r}_2,\mathbf{r}_1)\frac{1}{2m}\vec{\pi}(\mathbf{r}_1)G^{R(0)}(\mathbf{r}_1,\mathbf{r}_2)\right]. \quad (D11)$$

With the help of Eqs. (72), (B23), and (D7), one can transform the first term of Eq. (D11) to

$$-2n_{imp}U^2\frac{i\alpha(\mathbf{c}\cdot\mathbf{h}_\perp)}{2m\lambda^2}p\sum_n(n+1)\times G^R_{n+1}G^R_n[G^A_n e s_- - G^A_{n+1}e^* s_+] \quad (D12)$$

while the second term to

$$2n_{imp}U^2\frac{i\alpha(\mathbf{c}\cdot\mathbf{h}_\perp)}{2m\lambda^2}p\sum_n(n+1)[G^R_n e^* s_+ - G^R_{n+1}e s_-]G^A_{n+1}G^A_n. \quad (D13)$$

Taking into account the identities

$$G^{R(A)}_{n+1}G^{R(A)}_n=\frac{1}{\omega_c}(G^{R(A)}_{n+1}-G^{R(A)}_n), \quad (D14)$$

we obtain for the sum of Eqs. (D12) and (D13)

$$2n_{imp}U^2\frac{i\alpha(\mathbf{c}\cdot\mathbf{h}_\perp)}{2m\lambda^2\omega_c}p(e^*s_+-es_-)\sum_{n\geq 0}(n+1)\times(G^R_{n+1}G^A_{n+1}-G^R_nG^A_n). \quad (D15)$$

Here

$$\sum_{n\geq 0}(n+1)(G^R_{n+1}G^A_{n+1}-G^R_nG^A_n)=-\sum_{n\geq 0}G^R_nG^A_n=\frac{2\pi\tau}{p}\frac{N(\epsilon_F)}{1-i\omega\tau} \quad (D16)$$

and

$$(\mathbf{c}\cdot\mathbf{h}_\perp)(e^*s_+-es_-)=i(\mathbf{c}\cdot\mathbf{h}_\perp)(\hat{x}\sigma_y-\hat{y}\sigma_x)=-i\mathbf{c}\times\boldsymbol{\sigma}. \quad (D17)$$

Thus, on account of relation (69), we obtain

$$\mathbf{v}^{(sc)}_{(1.so)}=-\alpha\mathbf{c}\times\boldsymbol{\sigma}\frac{1}{1-i\omega\tau}\simeq-\alpha\mathbf{c}\times\boldsymbol{\sigma}(1+i\omega\tau). \quad (D18)$$

(3) In this item, we show that a correction bilinear in H_Z and H_{so} vanishes. First note that the term in the expansion of the Green' function bilinear in H_Z and H_{so} can be written in the form

$$G^{R(2)}_{(so,Z)}(\mathbf{r}_1,\mathbf{r}_2)=\int_{\mathbf{r}}[G^{R(1)}_{(so)}(\mathbf{r}_1,\mathbf{r})H_ZG^{R(0)}(\mathbf{r},\mathbf{r}_2)+G^{R(0)}(\mathbf{r}_1,\mathbf{r})H_ZG^{R(1)}_{(so)}(\mathbf{r},\mathbf{r}_2)], \quad (D19)$$

where $G^R_{(1.so)}$ is defined by Eq. (77). By making use of the explicit form of $G^R_{(1.so)}$ from Eq. (B20), the first and second terms of $G^{R(2)}_{(so,Z)}$ can be recast as

$$-\alpha\exp\left[\frac{1}{4}(z_1^*z_2-z_1z_2^*)\right]\frac{\omega_s(\mathbf{c}\cdot\mathbf{h}_\perp)}{2\lambda}\sum_n\sqrt{2(n+1)}[G^R_{n+1}(G^R_n)^2s_-(\mathbf{h}\cdot\tilde{\sigma})g^{(1b)}_n(z_1-z_2)+(G^R_{n+1})^2G^R_ns_+(\mathbf{h}\cdot\tilde{\sigma})g^{(1a)}_n(z_1-z_2)] \quad (D20)$$

and

$$-\alpha\exp\left[\frac{1}{4}(z_1^*z_2-z_1z_2^*)\right]\frac{\omega_s(\mathbf{c}\cdot\mathbf{h}_\perp)}{2\lambda}\sum_n\sqrt{2(n+1)}[(G^R_{n+1})^2G^R_n(\mathbf{h}\cdot\tilde{\sigma})s_-g^{(1b)}_n(z_1-z_2)+G^R_{n+1}(G^R_n)^2(\mathbf{h}\cdot\tilde{\sigma})s_+g^{(1a)}_n(z_1-z_2)]. \quad (D21)$$

The correction to the velocity bilinear in H_Z and H_{so} can be expressed through $G^{(2)}_{(so,Z)}$, $G^{(1)}_{(so)}$, and $G^{(1)}_{(Z)}$ as follows:

$$\mathbf{v}^{(sc)}_{(2.so,Z)}=n_{imp}U^2\int_{\mathbf{r}_1-\mathbf{r}_2}\Big\{G^{A(0)}(\mathbf{r}_2,\mathbf{r}_1)\frac{1}{2m}[\vec{\pi}^+(\mathbf{r}_1)+\vec{\pi}(\mathbf{r}_1)]G^{R(2)}_{(so,Z)}(\mathbf{r}_1,\mathbf{r}_2)+G^{A(2)}_{(so,Z)}(\mathbf{r}_2,\mathbf{r}_1)\frac{1}{2m}[\vec{\pi}^+(\mathbf{r}_1)+\vec{\pi}(\mathbf{r}_1)]G^{R(0)}(\mathbf{r}_1,\mathbf{r}_2)+G^A_{(1.Z)}(\mathbf{r}_2,\mathbf{r}_1)\frac{1}{2m}[\vec{\pi}^+(\mathbf{r}_1)+\vec{\pi}(\mathbf{r}_1)]G^R_{(1.so)}(\mathbf{r}_1,\mathbf{r}_2)+G^A_{(1.so)}(\mathbf{r}_2,\mathbf{r}_1)\frac{1}{2m}[\vec{\pi}^+(\mathbf{r}_1)+\vec{\pi}(\mathbf{r}_1)]G^{R(1)}_{(1.Z)}(\mathbf{r}_1,\mathbf{r}_2)\Big\}. \quad (D22)$$

Use of Eq. (D8) allows one to transform the first term in Eq. (D22) to the form

$$-n_{imp}U^2\frac{i\alpha(\mathbf{c}\cdot\mathbf{h}_\perp)\omega_s}{2m\lambda^2}p \times\sum_{n\geq 0}(n+1)\{[G^R_{n+1}(G^R_n)^2G^A_n es_- - (G^R_{n+1})^2G^R_nG^A_{n+1}e^*s_+](\mathbf{h}\cdot\boldsymbol{\sigma})+(\mathbf{h}\cdot\boldsymbol{\sigma})[(G^R_{n+1})^2G^R_nG^A_n es_- - G^R_{n+1}(G^R_n)^2G^A_{n+1}e^*s_+]\}, \tag{D23}$$

the second term to the form

$$-n_{imp}U^2\frac{i\alpha(\mathbf{c}\cdot\mathbf{h}_\perp)\omega_s}{2m\lambda^2}p\sum_{n\geq 0}(n+1)(\mathbf{h}\cdot\boldsymbol{\sigma})\{[G^R_{n+1}(G^A_{n+1})^2G^A_n es_- - (G^R_n)^2G^A_{n+1}(G^A_n)^2e^*s_+] + [G^R_{n+1}G^A_{n+1}(G^A_n)^2es_- - G^R_n(G^A_{n+1})^2G^A_ne^*s_+](\mathbf{h}\cdot\boldsymbol{\sigma})\}, \tag{D24}$$

the third term to the form

$$-n_{imp}U^2\frac{i\alpha(\mathbf{c}\cdot\mathbf{h}_\perp)\omega_s}{2m\lambda^2}p\sum_{n\geq 0}(n+1)(\mathbf{h}\cdot\boldsymbol{\sigma})G^R_{n+1}G^R_n[(G^A_n)^2es_- - (G^A_{n+1})^2e^*s_+], \tag{D25}$$

and the fourth term to the form

$$-n_{imp}U^2\frac{i\alpha(\mathbf{c}\cdot\mathbf{h}_\perp)\omega_s}{2m\lambda^2}p\sum_{n\geq 0}(n+1)[(G^R_{n+1})^2es_- - (G^R_n)^2e^*s_+](\mathbf{h}\cdot\boldsymbol{\sigma})G^A_{n+1}G^A_n. \tag{D26}$$

Thus, by dropping a common factor, we have for the sum of the all terms

$$\begin{aligned}(\mathbf{h}\cdot\boldsymbol{\sigma})&\sum_{n\geq 0}(n+1)\{[(G^R_{n+1})^2G^R_nG^A_n + G^R_{n+1}(G^A_{n+1})^2G^A_n + G^R_{n+1}G^R_n(G^A_n)^2]es_-\\ &-[G^R_{n+1}(G^R_n)^2G^A_{n+1} + G^R_nG^A_{n+1}(G^A_n)^2 + G^R_{n+1}G^R_n(G^A_{n+1})^2]e^*s_+\}\\ &+\sum_{n\geq 0}(n+1)\{[G^R_{n+1}(G^R_n)^2G^A_n + G^R_{n+1}G^A_{n+1}(G^A_n)^2 + (G^R_{n+1})^2G^A_{n+1}G^A_n]es_-\\ &-[(G^R_{n+1})^2G^R_nG^A_{n+1} + G^R_n(G^A_{n+1})^2G^A_n + (G^R_n)^2G^A_{n+1}G^A_n]e^*s_+\}(\mathbf{h}\cdot\boldsymbol{\sigma}).\end{aligned} \tag{D27}$$

By making use of the equalities

$$\omega_c\sum_n(n+1)(G^R_{n+1})^2G^R_nG^A_n = -\frac{2\pi\epsilon_F\tau^3}{p(1+i\omega\tau)^2}N(\epsilon_F),$$

$$\omega_c\sum_n(n+1)G^R_{n+1}(G^A_{n+1})^2G^A_n = -\frac{2\pi\epsilon_F\tau^3}{p(1+i\omega\tau)}N(\epsilon_F),$$

$$\omega_c\sum_n(n+1)G^R_{n+1}G^R_n(G^A_n)^2 = \frac{2i\pi\epsilon_F\tau^2}{p\omega_c}N(\epsilon_F)[(1+i\omega\tau)^{-2}-1], \tag{D28}$$

one can show that the sums standing in Eq. (D27) as a coefficient at $(\mathbf{h}\cdot\boldsymbol{\sigma})es_-$ vanishes. The same can be shown to be true with respect to other sums.

(4) In this item, we show that the term in $\mathbf{v}^{(sc)}_{(1.l)}$, which is proportional to α^3, namely,

$$\begin{aligned}\mathbf{v}^{(sc)}_{(3.so)} = n_{imp}U^2\int_{\mathbf{r}_1-\mathbf{r}_2} & G^{A(0)}(\mathbf{r}_2,\mathbf{r}_1)\frac{1}{2m}\\ &\times[\vec{\pi}^+(\mathbf{r}_1)+\vec{\pi}(\mathbf{r}_1)]G^{R(3)}(\mathbf{r}_1,\mathbf{r}_2)\\ &+G^{A(1)}(\mathbf{r}_2,\mathbf{r}_1)\frac{1}{2m}[\vec{\pi}^+(\mathbf{r}_1)+\vec{\pi}(\mathbf{r}_1)]G^{R(2)}(\mathbf{r}_1,\mathbf{r}_2)\\ &+G^{A(2)}(\mathbf{r}_2,\mathbf{r}_1)\frac{1}{2m}[\vec{\pi}^+(\mathbf{r}_1)+\vec{\pi}(\mathbf{r}_1)]G^{R(1)}(\mathbf{r}_1,\mathbf{r}_2)\\ &+G^{A(3)}(\mathbf{r}_2,\mathbf{r}_1)\frac{1}{2m}[\vec{\pi}^+(\mathbf{r}_1)+\vec{\pi}(\mathbf{r}_1)]G^{R(0)}(\mathbf{r}_1,\mathbf{r}_2)\end{aligned} \tag{D29}$$

vanishes. Here, for both the retarded and advanced functions,

$$\begin{aligned}G_{(3.so)}(\mathbf{r},\mathbf{r}') = \int_{\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3} & G^{(0)}(\mathbf{r},\mathbf{r}_1)H_{so}(\mathbf{r}_1)G^{(0)}(\mathbf{r}_1,\mathbf{r}_2)\\ &\times H_{so}(\mathbf{r}_2)G^{(0)}(\mathbf{r}_2,\mathbf{r}_3)H_{so}(\mathbf{r}_3)G^{(0)}(\mathbf{r}_3,\mathbf{r}').\end{aligned} \tag{D30}$$

In the explicit form

$$G_{(3.so)}(\mathbf{r}_1,\mathbf{r}_2) = -2(\mathbf{c}\cdot\mathbf{h}_\perp)\left(\frac{\alpha}{\lambda}\right)^3 \exp\left[\frac{1}{4}(z_1^* z_2 - z_1 z_2^*)\right] \times \sum_n \sqrt{2(n+1)^3}(G_{n+1}G_n)^2[g^{(1b)}(z_1-z_2)s_- + g^{(1a)}(z_1-z_2)s_+]. \quad \text{(D31)}$$

The explicit form of $G_{(1.so)}$ and $G_{(2.so)}$ are given by Eqs. (B20) and (B25). The first term in Eq. (D29) can be transformed to

$$-n_{imp}U^2\frac{2i(\mathbf{c}\cdot\mathbf{h}_\perp)}{m\lambda}\left(\frac{\alpha}{\lambda}\right)^3 p\sum_n (n+1)^2(G^R_{n+1}G^R_n)^2 \times[G^A_n es_- - G^A_{n+1}e^* s_+], \quad \text{(D32)}$$

the second term to

$$-n_{imp}U^2\frac{2i(\mathbf{c}\cdot\mathbf{h}_\perp)}{m\lambda}\left(\frac{\alpha}{\lambda}\right)^3 p\sum_n (n+2)(n+1) \times[G^R_{n+2}(G^R_{n+1})^2G^A_{n+1}G^A_n es_- - (G^R_{n+1})^2G^R_nG^A_{n+2}G^A_{n+1}e^* s_+], \quad \text{(D33)}$$

the third term to

$$-n_{imp}U^2\frac{2i(\mathbf{c}\cdot\mathbf{h}_\perp)}{m\lambda}\left(\frac{\alpha}{\lambda}\right)^3 p\sum_n (n+2)(n+1) \times[G^R_{n+2}G^R_{n+1}(G^A_{n+1})^2G^A_n es_- - G^R_{n+1}G^R_nG^A_{n+2}(G^A_{n+1})^2e^* s_+], \quad \text{(D34)}$$

and the fourth term to

$$-n_{imp}U^2\frac{2i(\mathbf{c}\cdot\mathbf{h}_\perp)}{m\lambda}\left(\frac{\alpha}{\lambda}\right)^3 p\sum_n (n+1)^2 \times[G^R_{n+1}es_- - G^R_n e^* s_+](G^A_{n+1}G^A_n)^2. \quad \text{(D35)}$$

Thus, by dropping a common factor, we have for the sum of all these terms

$$\sum_n (n+1)^2\{[(G^R_{n+1}G^R_n)^2G^A_n + G^R_{n+1}(G^A_{n+1}G^A_n)^2]es_- - [(G^R_{n+1}G^R_n)^2G^A_{n+1} + G^R_n(G^A_{n+1}G^A_n)^2]e^*s_+\} + \sum_n (n+2)(n+1) \times\{[G^R_{n+2}(G^R_{n+1})^2G^A_{n+1}G^A_n + G^R_{n+2}G^R_{n+1}(G^A_{n+1})^2G^A_n]es_- - [(G^R_{n+1})^2G^R_nG^A_{n+2}G^A_{n+1} + G^R_{n+1}G^R_nG^A_{n+2}(G^A_{n+1})^2]e^*s_+\}. \quad \text{(D36)}$$

This expression contains two terms proportional to es_- as well as two terms proportional to e^*s_+. Let us consider first the terms proportional to es_-. Use of the equalities

$$\sum_n \epsilon_n^2\begin{pmatrix}(G^R_{n+1}G^R_n)^2G^A_n\\ G^R_{n+1}(G^A_{n+1}G^A_n)^2\end{pmatrix} = \frac{\tau^4\epsilon_F^2}{(1+i\omega_c\tau)^2}\frac{2\pi i}{p}N(\epsilon_F)\begin{pmatrix}1\\-1\end{pmatrix} \quad \text{(D37)}$$

makes the sum that enters the first term proportional to es_-, vanish. Next consider the sum that enters the second term proportional to es_-. With the help of Eq. (D14), it can be transformed to

$$\frac{1}{\omega_c^2}\sum_n (n+2)(n+1) \times[(G^R_{n+2}-G^R_{n+1})(G^A_{n+1})^2 - (G^R_{n+1})^2(G^A_{n+1}-G^A_n)]. \quad \text{(D38)}$$

Now, use of the equalities

$$\sum_n \epsilon_n^2\begin{pmatrix}G^R_{n+2}(G^A_{n+1})^2\\ G^R_{n+1}(G^A_{n+1})^2\\ (G^R_{n+1})^2G^A_{n+1}\\ (G^R_{n+1})^2G^A_n\end{pmatrix} = \tau^2\epsilon_F^2\frac{2\pi i}{p}N(\epsilon_F)\begin{pmatrix}\frac{1}{(1+i\omega_c\tau)^2}\\ 1\\ -1\\ -\frac{1}{(1+i\omega_c\tau)^2}\end{pmatrix} \quad \text{(D39)}$$

makes the sum vanish too. Thus, all the coefficients that stand at es_- in Eq. (D36) vanish. The same is true with respect to coefficients standing at e^*s_+.

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